AIR TRAFFIC COMPLEXITY : TOWARDS INTRINSIC METRICS

D. Delahaye*, S. Puechmorel**
*** LOG(CENA) - 7, Avenue Edouard Belin 31055 TOULOUSE France**
(33) 5 62 17 41 79 / delahaye@recherche.enac.fr
**** ENAC Dept. MI - 7, Avenue Edouard Belin 31055 TOULOUSE France**
(33) 5 62 17 41 52 / puechmor@recherche.enac.fr

Abstract

This paper address the problem of measuring the air traffic complexity, given the observed positions and speeds of aircraft present in air space. Different studies have been conducted following various approaches like dynamic density and cognitive models. However, an intrinsic measure of the traffic complexity is still to be introduced in order to complete those previous works. Two classes of indicators are investigated. The first one uses geometrical properties in order to build a new complexity coordinate system in which sector complexity evolution through time is represented. The second one uses a representation of air traffic as a dynamical system, yielding, through the topological Kolmogorov entropy, an intrinsic measure of complexity.

1 Introduction

Nowadays, when an air sector is said to be overloaded, this means that the number of aircraft which have crossed this sector during the last current hour has reached a limit called the capacity. This number of aircraft (usually expressed in "flights per hours") tries to summarize the level of operational congestion in order to apply some regulations when it reaches the capacity. When an operational sector is observed during several days it can be noticed that sometimes, the controller in charge of this sector accept more aircraft than the actual capacity and at some other period refuse some traffic even if the capacity has not been reached. Thus, the feelling of the controller about the complexity of the traffic for which he is in charge can not be summarized by a simple number of aircraft even if those two quantities (number of aircraft and complexity) are correlated. This feelling, called workload, is much more complex and is related to many factors for which some are quantitative and some others are qualitative. From the modeling point of view, it seems too difficult to define a mathematical formulation of the real controller workload because of its complexity. The goal of this study is to synthesize a traffic complexity indicator in order to better quantify the congestion in an air sector, which will be more relevant than a simple number of aircraft. More precisely, our objective is to build some metrics of the intrinsic complexity of the distribution of traffic in the airspace.

Those metrics are relevant for many applications in the air traffic management area. For instance, when a sectoring is designed [1], the sectors have to be balanced from the congestion point of view and for the time being,only the number of aircraft is used to reach this objective . Another example where a congestion metric is needed is the traffic assignment [2], [3] for which an optimal time of departure and a route are searched in order to reduce the congestion in the air sector then a more precise measure of the congestion is needed to reach this goal. This complexity indicator may also be used to design new air networks, for the dynamic sectoring concept, to define some Free Flight areas etc ... .

Complexity metrics would enable to qualify and quantify the performance of the Air Traffic services providers and permit a more objective consultation between airlines and providers.

Some previous efforts have been done to define some air traffic complexity indicators and our approach may be used to extend those works. The main improvement is based on the measure of the disorder of the speed vector field in the three dimension space.

The first part of this paper presents the previous related works which were manly done at NASA ames and at Wyndemere Inc. In the same part, some other classical complexity indicators such as information theoretical entropy, time-space frequency representation (wavelet) are presented and the reasons why they are not relevant for our problem are given. The second part presents a geometrical approach based on the relative speed vectors and on the relative distances. It proposes a new representation of the set of aircraft in a time-complexity space. The third part describe a complexity indicator based on the dynamic systems theory using the concept of topological entropy (Kolmogorov-entropy) which produce a good measure of the actual disorder of the speed vector in an air sector even if the number of aircraft is small. Finally, the fourth part presents a validation process in order to adjust this in different associated parameters.

2 Previous related works

The airspace complexity is related with both the structure of the traffic and the geometry of the airspace. Different efforts are underway to measure the whole complexity of the airspace.

Windemere inc [4] proposed a measure of the perceived complexity of an air traffic situation. This measure is related with the cognitive workload of the controller with or without the knowledge of the intents of the aircraft. The metric is human oriented and is then very subjective. Laudeman et al from NASA [5] have developed a metric called « Dynamic Density » which is more quantitative than the previous one and is based on the flow characteristics of the airspace. The « Dynamic Density » is a weighted sum of the traffic density (number of aircraft), the number of heading changes (> 15°), the number of speed change (> 0.02 Mach), the number of altitude changes (> 750 ft), the number of aircraft with 3-D Euclidean distance between 0-5, the number of altitude changes (> 750 ft), the number of aircraft with 3-D Euclidean distance between 0-25, the number of conflicts predicted in 0-25 nautical miles, the number of conflicts predicted in 25-40 nautical miles and the number of conflicts predicted in 40-70 nautical miles. The parameters of the sums have been adjusted by showing different situation of traffic to several controllers. Finally, B.Sridhar (NASA 1998) [6] developed a model to predict the evolution of this metric in the near future.

The previous models do not take into account the intrinsic traffic disorder which is related to the complexity. The goal of the present work is to extend those works with a new metric of desorder.

In information theory, the disorder is measured with the entropy E. If the $p(x)$ is the probability density function of the variable X, the associated entropy is given by:

$$E = -\int_{-\infty}^{+\infty} p(x).\log\{p(x)\}dx$$

If the speed vector fields are considered in the high-density areas, it is possible to define the two-dimensional speed vector distribution: $f(v_x, v_y)$.

The associated entropy is given by:

$$E = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(v_x, v_y)\log\{f(v_x, v_y)\}dv_x dv_y$$

This statistical measure of the disorder is relevant only if the probability distribution has a real meaning. From the airspace point of view, this disorder metric is computed at time *t* and need many speed samples in order to be relevant from the statistical point of view. Unfortunately, when a sector is observed at time *t*, the maximum number of aircraft is 30, which is not enough to synthesis a good probability density function.

In order, to avoid this drawback, we try to work on the relative speeds between aircraft instead of the absolute speeds. But from the information theory, it can be shown that there is no information gain when the variable are mixed with a linear combination which is the case for the relative and the absolute speeds. This mean that when the number of samples is artificially increased by a linear weighted sum of the former ones, the level of information given by this sample is the same. So, the entropy calculation on the relative speeds has the same drawback as the original one.

Afterward, another approach based on the wavelets has been developed to give a representation of the traffic in a new coordinate system. This approach takes a picture on the traffic at time *t* and computes a wavelet transform of the space distribution and speed vector distribution of aircraft. With this transform, it is very easy to identify the high/low density areas and the high/low disorder cluster.

The problem comes from the wavelet transform computation, which needs a minimum texture density in order to produce relevant result. As for the entropy calculation, the number of aircraft being two small on this «picture », the transform produces only noise.

None of the former indicators being satisfactory, two new indicators of the complexity of traffic, which avoid the previous drawback, have been developed. The first one is based on the geometry of the traffic and the second one uses the Kolmogorov entropy coming from the theory of the dynamical system.

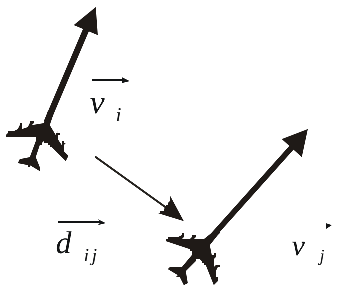
Those indicators are now presented in the two following section.

3 Geometrical approach

This approach is based on the properties of the relative positions and the relative speeds of aircraft in a sector. When a set of aircraft is considered in a sector, it is possible to identify different areas for which the structure of traffic is different. For example, it is possible to identify some high density zones and clusters of traffic with strong disorder. This identification is done by our brain which investigate the different structure and is able to regognize structure symmetries. The current approach, propose some metrics in order to quantify this feelling of disorder and produces a new representation for which each aircraft may be assigned to a point in a complexity coordinate system.

When two aircraft are considered, it is possible to define their relative distance and their relative speed.

The relative distance is given by:



$$\overrightarrow{d_{ij}} = \overrightarrow{P_j} - \overrightarrow{P_i}$$

In the same way, the relative speed is define as follow:

$$\overrightarrow{v_{ij}} = \overrightarrow{v_j} - \overrightarrow{v_i}$$

In the following $\left\|\overrightarrow{d_{ij}}\right\|$ will represent the relative distance vector $\overrightarrow{d_{ij}}$.

It can be easily shown that the derivative of this norm is given by:

$$\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt} = \frac{\overrightarrow{d_{ij}}.\overrightarrow{v_{ij}}}{\left\|\overrightarrow{d_{ij}}\right\|}$$

Where "." is the scalar product.

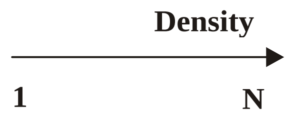
This norm will now help us to define our aircraft density measure:

$$Dens(i) = 1 + \sum_{\substack{j=1 \\ j\neq i}}^{N} e^{-a\frac{\left\|\overrightarrow{d_{ij}}\right\|}{R}}$$

Where

- i is the current aircraft for which the local density is computed
- N is the whole number of aircraft
- a is a weighted coefficient
- R is a neighborhood distance

The exponential function is here to give much more



weight to the aircraft, which are close to the current aircraft. Then, each aircraft is located on the density axe on a position between 1 to N.

Another axis will represent the level of "disorder" and a distinction will be made between the positive part and the negative one. The part above the x coordinate represents the level of divergence. The divergence between two aircraft measure how fast they move away from each other. The global divergence of an aircraft "i" is the a weighted sum of all the divergence between pairs of aircraft:

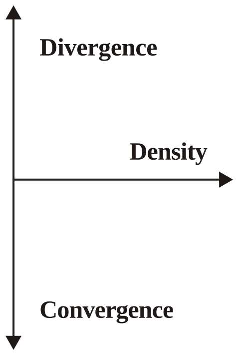
$$Div(i) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \left|\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt}\right| . 1_{R^+}\left\{\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt}\right\} . e^{-a\frac{\left\|\overrightarrow{d_{ij}}\right\|}{R}}$$

Where 1_{R^+} is the indicatrice function on R^+

In the same way, it is possible the define the level of convergence of an aircraft by taking into account only the aircraft which get closer from each other.

$$Conv(i) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \left|\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt}\right| . 1_{R^-}\left\{\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt}\right\} . e^{-a\frac{\left\|\overrightarrow{d_{ij}}\right\|}{R}}$$

Our system has now two axis x and y, which represent respectively the density and the levels of convergence and divergence.



Finally a third axis (z) will support the sensitivity of the relative distance to the classical maneuver such as speed and heading changes in case of convergence.
Two different metrics may be used to describe this indicator depending of the user objective.

The first one is related to the gradient of the relative distance. This indicator measure the change in term of relative distance when small modification is applied to the speeds and the headings of the aircraft involved.

$$\left\|\nabla_{\|\vec{d_{ij}}\|}\right\| = \left\| \begin{pmatrix} \frac{\partial \|\vec{d_{ij}}\|}{\partial v_j} \\ \frac{\partial \|\vec{d_{ij}}\|}{\partial v_i} \\ \frac{\partial \|\vec{d_{ij}}\|}{\partial q_j} \\ \frac{\partial \|\vec{d_{ij}}\|}{\partial q_i} \end{pmatrix} \right\|$$

$$\left\|\overrightarrow{\nabla_{\|\vec{d_{ij}}\|}}\right\| = \left\| \begin{pmatrix} a_x \sin(q_j) + a_y \cos(q_j) \\ -\left(a_x \sin(q_{ij}) + a_y \cos(q_{ij})\right) \\ v_j\left(-a_x \cos(q_j) + a_y \sin(q_j)\right) \\ v_i\left(a_x \cos(q_i) - a_y \sin(q_i)\right) \end{pmatrix} \right\|$$

where

$$a_x = \frac{\Delta_x}{\sqrt{\Delta_x^2 + \Delta_y^2}} \quad a_y = \frac{\Delta_y}{\sqrt{\Delta_x^2 + \Delta_y^2}}$$

$$\Delta_x = x_j - x_i \quad \Delta_y = y_j - y_i$$

The sensitivity indicator is then given by :

$$Sd_-(i) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left\|\overrightarrow{\nabla_{\|\vec{d_{ij}}\|}}\right\| . 1_{R^-}\left\{\frac{d\|\vec{d_{ij}}\|}{dt}\right\} e^{-a\frac{\|\vec{d_{ij}}\|}{R}}$$

$$Sd_+(i) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left\|\overrightarrow{\nabla_{\|\vec{d_{ij}}\|}}\right\| . 1_{R^+}\left\{\frac{d\|\vec{d_{ij}}\|}{dt}\right\} e^{-a\frac{\|\vec{d_{ij}}\|}{R}}$$

This indicator is very sensitive to the angle of crossing and is maximum for a face to face convergence

The second indicator is related to the sensitivity of the conflict duration with the speed and heading modifications.
When two aircraft are close from each other and are in convergence, the time of conflict is given by:

$$t_c = \frac{\|\vec{P_j} - \vec{P_i}\|}{\|\vec{v_j} - \vec{v_i}\|} = \frac{\|\vec{d_{ij}}\|}{\|\vec{v_{ij}}\|} = \frac{\|\vec{d_{ij}}\|}{\sqrt{\Delta v_x^2 + \Delta v_y^2}}$$

$$\Delta v_x = v_j \sin(q_j) - v_i \sin(q_i)$$
$$\Delta v_y = v_j \cos(q_j) - v_i \cos(q_i)$$

We have

$$\overrightarrow{\nabla t_c} = \begin{pmatrix} \frac{\partial t_c}{\partial v_j} \\ \frac{\partial t_c}{\partial v_i} \\ \frac{\partial t_c}{\partial q_i} \\ \frac{\partial t_c}{\partial q_j} \end{pmatrix}$$

$$\frac{\partial t_c}{\partial v_j} = \|\vec{d_{ij}}\| . \frac{\Delta v_x \sin(q_j) + \Delta v_y \cos(q_j)}{\left(\|\vec{v_{ij}}\|\right)^3}$$

$$\frac{\partial t_c}{\partial v_i} = \|\vec{d_{ij}}\| . \frac{-\Delta v_x \sin(q_i) + \Delta v_y \cos(q_i)}{\left(\|\vec{v_{ij}}\|\right)^3}$$

$$\frac{\partial t_c}{\partial q_i} = \left\| \overrightarrow{d_{ij}} \right\| . \frac{-\Delta v_x \cos(q_j) + \Delta v_y \sin(q_j)}{\left(\left\| \overrightarrow{v_{ij}} \right\| \right)^3}$$

$$\frac{\partial t_c}{\partial q_j} = \left\| \overrightarrow{d_{ij}} \right\| . \frac{\Delta v_x \cos(q_i) - \Delta v_y \sin(q_i)}{\left(\left\| \overrightarrow{v_{ij}} \right\| \right)^3}$$

The sensibility indicator

$$St_-(i) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left\| \overrightarrow{\nabla_{t_c}} \right\| . 1_{R^-} \left\{ \frac{d \left\| \overrightarrow{d_{ij}} \right\|}{dt} \right\} e^{-a \frac{\left\| \overrightarrow{d_{ij}} \right\|}{R}}$$

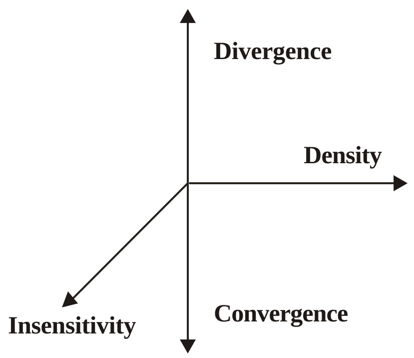
$$St_+(i) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left\| \overrightarrow{\nabla_{t_c}} \right\| . 1_{R^+} \left\{ \frac{d \left\| \overrightarrow{d_{ij}} \right\|}{dt} \right\} e^{-a \frac{\left\| \overrightarrow{d_{ij}} \right\|}{R}}$$

Due to the fact that a convergent situation with a high sensitivity is better than a convergent situation with a low sensitivity, the induced complexity will be higher in the later than in the former. In order to have an homogeneous coordinate system, the insensitivity is define as follow:

$$ISd_-(i) = \frac{1}{e + Sd_-(i)}$$

$$ISd_+(i) = \frac{1}{e + Sd_+(i)}$$

Our complexity coordinate system has now three coordinates: density, divergence/convergence and insensitivity.



The complexity of a given traffic situation will be represented by a path in this new coordinate system. Computation of this indicator on some representative traffic situation are given at the end of this paper and is compared with the dynamical system approach that is now presented.

4 Dynamical systems and complexity

4.1 Motivation

Through the geometric approach, a dynamic representation of the traffic in terms of potential conflicts and aircraft density was defined. However, the evolution of the set of aircrafts as a whole is not taken into account. In many cases, most of the perceived complexity arises from the observation of the history of the traffic.

It appears thus that the dynamical aspect of air traffic is of utmost importance in the definition of a complexity metric. Several smoothing in time procedures can be designed using a geometric complexity measure sampled periodically, but will lack both intrinsic character and theoretical foundation.

A more promising approach is to model the history of air traffic as the evolution of a hidden dynamical system such that aircrafts correspond to pointwise observations. Of course, there is an infinite number of dynamical systems that may fit the requirement that observed aircrafts trajectories be system trajectories as well. Practically, additional assumptions will be made on the representing dynamical system so that the model can be uniquely chosen.

Having the underlying dynamical system at hand, it exists a measure of its intrinsic complexity : the topological entropy, which is the root of the mathematical ergodic theory.

We will now define the topological entropy and discuss its adaptation to our problem, ending with a new air traffic complexity metric.

4.2 Topological Entropy

In the following, and otherwise noted, X will stand for a compact metric space and $T: X \longrightarrow X$ for a continuous map, so that the couple (X, T) defines a dynamical system. Recall that the notation T^n means the operator T iterated n times, and that the trajectory starting at $x \in X$ is the net $(T^n x)_{x \in N}$. Let Ω be the set of all finite open covers of X. Ω is a partially ordered set with the inclusion as order relation. The refinement of two finite open covers $a, b \in \Omega$ is the finite open cover denoted by $a \vee b$ and such that :

$$a \vee b = \{A_i \cap B_j, A_i \in a, B_j \in b, A_i \cap B_j \neq \emptyset\}$$

It is clear that :

$$card(a \vee b) \leq card(a) card(b)$$

For $a \in \Omega$, the finite open cover $T^{-1}(a)$ is defined by :

$$T^{-1}(a) = \{T^{-1}(A_i), A_i \in a\}$$

Note that the direct image of a finite open cover is not a finite open cover unless $T(X) = X$ and T is an open map.

Definition 4.1

The topological entropy of a cover $a \in \Omega$, denoted by $H(a)$, is defined by :

$$H(a) = \inf_{b \subset a} \ln(card(b))$$

Definition 4.2

The topological entropy of the map T relative to the cover $a \in \Omega$ is the quantity:

$$h(T, a) = \lim \sup_{n \to +\infty} \frac{1}{n} H\left(\vee_{i=0}^{n-1} T^{-i} a \right)$$

Definition 4.3

The topological of the map T is :

$$h(T) = \sup_{a \in \Omega} h(T, a)$$

Remark

The quantity $h(T, a)$ is well defined since it can be easily proved that

$$H(a \vee b) \leq H(a) + H(b)$$

since

$$card(a \vee b) \leq card(a) card(b)$$

So we have the inequality :

$$\frac{1}{n} H\left(\vee_{i=0}^{n-1} T^{-i} a \right) \leq H(a)$$

proving that $h(T, a) < +\infty$

Definition 4.4

A open finite cover a is a generator for the map T if $\forall e > 0; \exists n_0$ such that the cover $\vee_{i=0}^{n_0} T^{-i} a$ has no element of diameter greater than e .

The interest of generators lies in the following proposition :

Proposition 4.1

Let a be a generator for T. Then $h(T, a) = h(T)$.

This provides a practical way of computing the entropy of a map, once a generator has been found.

4.3 Air traffic as a dynamical system

Recall that we are seeking after a well defined dynamical system model (X, T) such that the observed aircrafts trajectories are trajectories of this dynamical system. Once the model has been found, the proposed complexity metric will be the entropy of the map T.

First of all, let us assume that we have observed during N steps the positions and speeds of M

aircrafts, so that the input parameters for the model are M N-uples of the form :

$$(x_i, q_i)_{i=1\cdots N}$$

with x_i, q_i respectively the position and speed vector of the aircraft at step i. First of all, the (compact) space X needs to be defined. Basically, there is two ways of constructing X :

- Enclose the observed region into a rectangle, then identify sides of the bounding rectangle to give a torus. Resulting space X is compact, but has no operational counterpart (aircraft crossing the boundary of the observed region will reappear in the domain at another boundary).
- Take for X the one point compactification of R^3.

The second choice has been retained, due to its operational soundness, as well its good theoretical properties.

Next step is the construction of the continuous map *T*. The most simple way of having uniqueness in the definition is to require the local vector fields associated with T to be of minimum curvature, which yields a piecewise polynomial approximation. In a first approach, one can use a linear interpolation, which gives satisfactory results at a low computational cost.

4.4 Topological entropy of air traffic

Since the operator T is can be constructed by the previous procedure, its now only a matter of computational efficiency to obtain the topological entropy of the dynamical system. One must anyway be careful about the fact that since trajectories will generally intersect with each other, entropy must be computed locally. Experimental study of this indicator is in progress ; however, some point have to be stressed now:

- The topological entropy is an intrinsic measure of the complexity of the geometry of the traffic. Traffic with crossing trajectories has higher entropy.
- Although relying on a dynamical system, it does not measure conflicts between aircrafts, but between flows.

4.5 Including conflicts

As previously mentioned, topological entropy measures flux complexity but not individual aircrafts interactions. In fact, depending on the application of the indicator, both approaches can be relevant. However, probing conflicts difficulty is a major concern in ATC systems, so that there is a need for a complexity measure adapted to it. Basically, there is two approaches relating entropy to conflict probing :

- Compute conflict probability in time and space and then compute entropy associated to the resulting distribution. Since there are too few samples in general to accurately construct an entropy estimator, one needs a stochastic model of air traffic and conflicts. This approach is yet to be investigated, but may give some interesting results since it is known from operational experiences that a given traffic situation has a high complexity precisely when conflicts are close in time and space (high density and frequent conflicts). Be careful that this estimator has an opposite behavior compared with the topological entropy : high values indicate a nearly uniform distribution of the conflicts in time and space, which corresponds to a low complexity.
- Construct an extension of the topological entropy with respect to a time-space metric. This is the most satisfactory way of taking conflicts into account, and this is the one that

has been used for computed entropy on test situations below.

5 Calibration process

The two previous approaches have several degree of freedoms, which have to be fixed with operational data sets. To reach this goal, as for the dynamic density adjustment, a set of traffic situations will be presented to several controller teams who will compare them in term of complexity by answering if situation "A" is more complex than situation "B" for all pair of samples. From those answers, it will be possible to determine the most discriminant parameters of the models. Finally, the value of those parameters will be adjusted by a statistical procedure.

6 Results

Four characteristic simulated traffic situations have been investigated : a random flow (41 aircraft with random initial position and speed), a parallel flow with no conflicts, a right angle crossing and a low angle crossing.

Geometric complexity representation is drawn for each of the previous situation. The curves Density-Convergence and unsensitivity-time are drawn on the same graph.

We observe that the fully organised situation (parallel flow) does not generate complexity at all, either from the geometrical or dynamical system point of view (0 entropy). Next one in complexity is right angle, then random flow, then the most complex of the four, namely the low angle crossing. It may be noted that both indicators give the same order in complexity.

7 Conclusion

As previously pointed out, the present definition of sector capacity is not sufficient to describe the real complexity of the airspace. The number of aircraft in an air sector does not encompass the strong complexity of the mental workload involved in the control process, even if those two entities are very dependent. From the mathematical point of view, it seems impossible to build an exact model of this control workload since it depends of many parameters which are both qualitative and quantitative. Between those two extremes, different levels of detail may be find to model the control workload.

Several efforts are underway, but all of them do not take into account the intrinsic disorder of the traffic which is critical in the definition of the airspace complexity.

Two new approaches have been proposed to refine those previous works.

The first one describes an air traffic complexity indicator based on the structure and the geometry of the traffic. This indicator produce a point representng the sector complexity in a new coordinate system.

The second approach is based on the dynamic system theory and uses the Kolmogorov-Entropy to measure the global disorder of the aircraft system when it evolves with time.

Those two new indicators may be used to improve and upgrade the concept of dynamic density.

8 Figures

8.1 Kolmogorov Entropy

Traffic Situation	**Associated Entropy**
Random flow	8274
Parallel flow	0
Right angle crossing	64173
Low angle crossing	487267

8.2 Geometrical complexity

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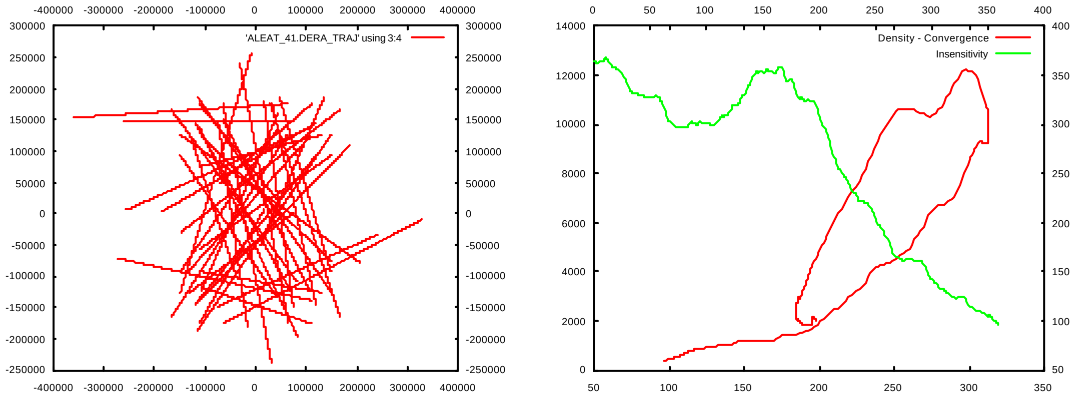
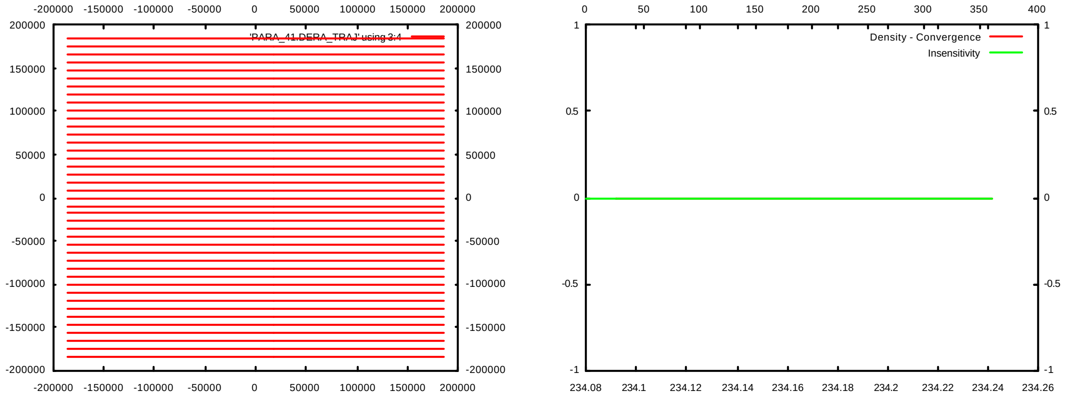
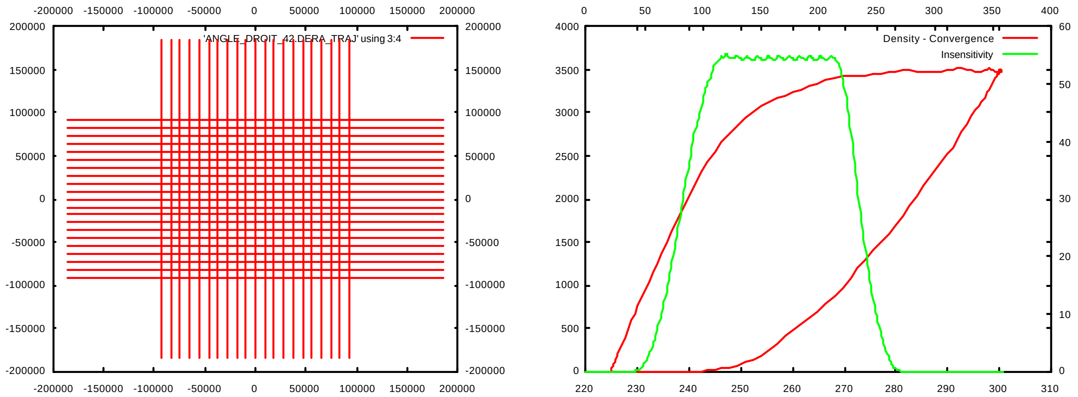


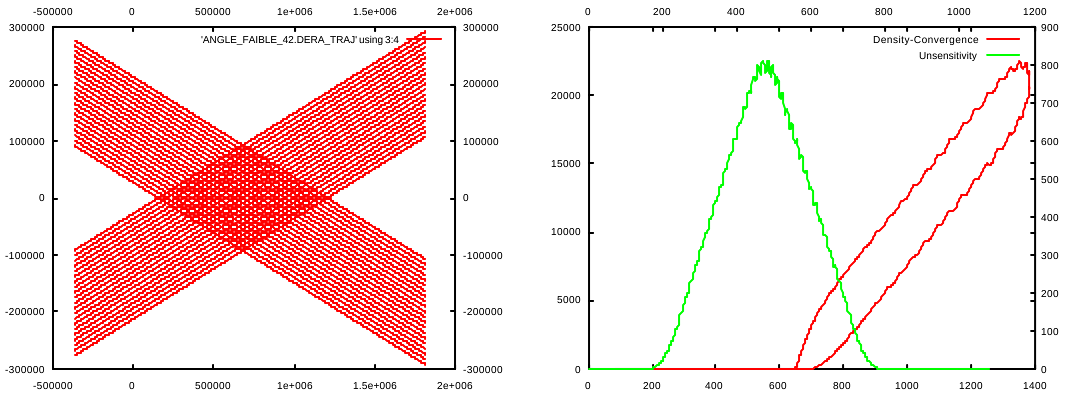
Random speed and position



Parallel flow



Right angle crossing



Low angle crossing

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