

Draft Paper: An Airspace Planning Model for Selecting Flight-Plans Under Workload, Safety, and Equity Considerations

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Summary

In this paper, we present an *Airspace Planning Model (APM)* that has been developed for use under future airspace scenarios. Given a set of flights for a particular time horizon, along with (possibly several) alternative flight-plans for each flight that are based on delays and diversions due to *special-use airspace* (SUA) restrictions or weather considerations, this model prescribes a set of flight-plans to be implemented. The model formulation seeks to minimize a delay and fuel cost based objective function, subject to the restrictions that each flight is assigned one of the designated flight-plans, and that the resulting set of flight-plans satisfies certain specified workload, safety, and equity criteria. These requirements ensure that the workload for air-traffic controllers in each sector is held under a permissible limit, that any potential conflicts which may occur are routinely resolvable, and that the various airlines involved derive equitable levels of benefits from the overall implemented schedule. In order to solve the resulting 0-1 mixed-integer programming problem more effectively using commercial software (CPLEX-MIP), we explore the use of various facetial cutting planes and reformulation techniques designed to more closely approximate the convex hull of feasible solutions to the problem. Computational results will be reported on several scenarios based on actual flight data obtained from the *Federal Aviation Administration* (FAA) in order to demonstrate the efficacy of the proposed approach for *air traffic management* (ATM) purposes.

1 Introduction

With the advent of the *Free-Flight* paradigm of airline operations, and with the widespread

use of the *Reusable Launch Vehicle* (RLV) technology, many challenges arise in the context of airspace planning. The *Free-Flight* paradigm permits airlines to compose more cost-effective routes for its flights rather than traverse paths between designated way-points from various origins to different destinations. In addition, it is anticipated that the RLV technology will add as many as 1,200 shuttle launches in the next decade (AST, 1998). This significant increase in space transportation traffic in the future is due to the ability of an RLV launch to perform the same function at a fraction of the cost as compared with a space shuttle voyage. Given that each such RLV launch involves the cordoning of large regions of airspace around the spaceport in order to ensure the safe transit of the launch vehicle, this can result in a severe disruption of commercial traffic that goes well beyond present-day levels.

This future airspace scenario requires new approaches for providing operational and policy guidelines for safely and economically routing and scheduling commercial air-traffic. Such guidelines include limiting the number of aircraft traversing each given sector of the airspace at any point in time to within a manageable workload level for the corresponding sector's *air-traffic control* (ATC) capabilities. We may also require that ATC operators never have to resolve more than one potential conflict within a designated segment of time, and that any such potential conflict that is not expressly prohibited is easily resolvable. Furthermore, due to the sharp increase in space transportation, many flights will be delayed as a result of detoured routing around the ensuing *Special-Use Airspace* (SUA) restrictions. This feature creates the need for the airlines and the *Federal Aviation Administration* (FAA) to jointly ad-

dress the efficient and equitable management of these delays.

Prompted by the foregoing considerations, we develop a 0-1 mixed-integer programming model in this paper, called the *Airspace Planning Model (APM)*, in response to FAA's need for a collaborative decision-making tool. Given a set of flights to be scheduled over a planning horizon, along with several alternative flight-plans for each flight, this model selects a set of flight-plans that minimizes a delay and fuel cost based objective function, such that each flight is executed via a designated flight-plan, while satisfying certain specified workload, safety, and equity restrictions. Within the framework of generating our model, we utilize two particular submodels developed by Sherali, et al. (1999). These two submodels are the *Airspace Sector Occupancy Model (AOM)* which determines the entering and exiting times of flights traveling through certain 3-dimensional nonconvex regions of airspace called *sectors*, and the *Aircraft Encounter Model (AEM)* which detects all possible pairwise conflicts between flight-plans, providing detailed statistics regarding the severity and geometry of each conflict. The information garnered from these two models are used to generate the various constraints for the Airspace Planning Model developed herein.

Falker (1999) provides an in-depth review of current National Airspace policies, under which air and space traffic are kept separate. Future airspace scenarios are examined in which space transportation has significantly increased. Under this scenario, several alternative modes of operations are examined using a discrete event simulation process with respect to their safety, efficiency, equity, and ease of implementation. Detailed summaries on various ex-

isting air traffic management models may be found in Odoni, et al. (1997), and in a concept paper by the Federal Aviation Administration/Eurocontrol (1998). The *Analytic Blunder Risk Model (ABRM)* is an analytic/probabilistic model which estimates collision risk between two aircraft, when one aircraft strays from its intended course and the other must avoid it. The *Traffic Organization and Perturbation Analyzer (TOPAZ)* evaluates the safety of any given operational concept by utilizing a suite of simulation software packages. TOPAZ differs from other fast-time simulation packages by allowing probabilistic deviations from normal operating conditions. The *National Airspace Resource Investment Model (NARIM)* is an analytical tool developed for the purposes of FAA and *National Aeronautics and Space Administration (NASA)* in order to evaluate future airspace scenarios. NARIM uses several analytical tools along with data collected from aircraft operations to provide a broad spectrum of analyses pertaining to alternative scenarios, operational issues, system-wide performance, and the exploration of various new concepts of operations.

2 The Airspace Planning Model

Consider a planning horizon H , and suppose that we are given a set of flights $i \in M \equiv \{1, \dots, m_H\}$ covering this horizon that are relevant to a certain region of airspace. For each flight $i \in M$, let P_i be the set of possible flight-plans p composed of departure and anticipated arrival times, along with trajectory and cruising altitude information when traversing a route between the corresponding origin-destination pair. Note that there will typically exist some pre-

ferred departure time for each flight, along with some alternative (discrete) departure schedules. For each departure time, given the existing state of special-use airspace restrictions, one or more flight-plans corresponding to different trajectories and/or cruising altitudes could be generated using some commercial package. Given any such combination (i, p) , $i \in M$, $p \in P_i$, we can compute a cost factor c_{ip} for adopting plan p for flight i . (SIMMOD (Federal Aviation Administration, 1993), for example, can be used for this purpose.) This cost would reflect fuel expended, delay costs, as well as penalties or benefits (rewards or negative penalties) based on the relative desirability of the particular flight-plan.

Accordingly, defining the *decision variables* $x_{ip} \forall i \in M, p \in P_i$ as $x_{ip} = 1$ if plan $p \in P_i$ is adopted, and 0 otherwise, we can formulate a total system-based objective function to

$$\text{Minimize } \sum_{i \in M} \sum_{p \in P_i} c_{ip} x_{ip}. \quad (1)$$

The constraints would include the selection of a plan for each flight as specified by

$$\sum_{p \in P_i} x_{ip} = 1 \quad \forall i \in M, \quad (2)$$

as well as certain equity, workload, and conflict resolution restrictions that are discussed next.

2.1 Equity Constraints:

Suppose that there are some F airline firms involved in this study, indexed by $f = 1, \dots, F$. In the process of selecting flight-plans based on (1) and (2) (in addition to workload and conflict resolution constraints as described in the sequel), we would also like to achieve a degree of equity among the airline firms. For each firm

$f = 1, \dots, F$, let us define a *measure of ineffectiveness* M_f as the average cost per flight given by

$$M_f = \frac{1}{n_f} \sum_{(i,p) \in A_f} c_{ip} x_{ip} \quad (3a)$$

where

$$A_f \equiv \{(i, p) : \text{flight } i \text{ belongs to firm } f\}, f = 1, \dots, F, \quad (3b)$$

and where n_f is the number of flights that belong to firm f , for $f = 1, \dots, F$. Accordingly, defining the equity variables x_l^e and x_u^e to respectively represent the lower and upper limits of the range of variation for the ineffectiveness measures M_f , $f = 1, \dots, F$, where x_u^e is restricted to be no more than some specified value ν_e , we can model equity via the following mechanism.

Include the following restrictions within the constraints:

$$x_l^e \leq \frac{1}{n_f} \sum_{(i,p) \in A_f} c_{ip} x_{ip} \leq x_u^e \quad \forall f = 1, \dots, F \quad (4)$$

$$x_l^e \geq 0, \quad x_u^e \leq \nu_e. \quad (5)$$

Include the following terms within the objective function:

$$(\text{Minimize}) \dots + \mu_e (x_u^e - x_l^e) + \mu_u^e x_u^e \quad (6)$$

where μ_e is a (commensurate) penalty per unit of variation in the measures M_f , $f=1, \dots, F$, and μ_u^e is a (commensurate) penalty for the maximum incurred measure of ineffectiveness.

2.2 Workload Constraints:

Consider the total collection of flight-plans $\bigcup_{i \in M} P_i$. Jointly, these plans involve traversals between certain pairs of fixes, as well as free-flight cruises between designated pairs of fixes, at various specified altitudes. Let us consider a segmentation of the airspace into *sectors* as defined by FAA (these are generally nonconvex polygons, lifted into the vertical dimension) and let S be the set of all sectors involved with the collection of flight-plans $\bigcup_{i \in M} P_i$.

Define the *workload* for a sector at any point in time to be the number of aircraft that are resident within that sector at the given instant of time. To characterize this workload we can examine the occupancy durations of the flights $i \in M$ within each sector $s \in S$, over the horizon H . The model AOM of Sherali, et al. (1999) provides this information by constructing a Gantt chart of flight-plan occupancy intervals for each sector.

Naturally, for any sector, whenever we have an overlap of such occupancy durations, we would have a potential increase in workload. Hence, for each sector $s \in S$, let $k = 1, \dots, K_s$ index the collection of maximal overlapping sets C_{sk} of flight-plans (i, p) , where an overlapping set of flight-plans is called maximal if it is not a strict subset of another overlapping set. An efficient algorithm for determining these sets is described in Sherali and Brown (1994). Let us now define the variable n_s to represent the maximum number of overlapping flights within each sector $s \in S$, and let us bound this variable on a suitable interval $[1, \bar{n}_s]$, and furthermore, penalize its value in the objective function using a penalty factor that increases nonlinearly in an appropriate fashion with an increase in workload. The motivation here is that

if the maximum number of aircraft being simultaneously monitored in a sector increases from one to three, for example, the associated penalty should likely more than triple. Hence, let us define the binary variables $y_{sn} \forall s \in S, n = 1, \dots, \bar{n}_s$ as $y_{sn} = 1$ if the maximum workload in sector s is n , and 0 otherwise, and let μ_{sn} be the associated penalty for having $y_{sn} = 1$. We assume that

$$\mu_{s2} \geq \mu_{s1}, \text{ and } \mu_{sj} \geq 2\mu_{s(j-1)} - \mu_{s(j-2)} \quad \forall j = 3, \dots, \bar{n}_s. \quad (7)$$

Note that the condition (7) implies that

$$0 \leq (\mu_{s2} - \mu_{s1}) \leq (\mu_{s3} - \mu_{s2}) \leq \dots \leq (\mu_{s\bar{n}_s} - \mu_{s\bar{n}_s-1}) \quad (8)$$

and imparts a convex nondecreasing penalty structure. For example, we might have $0 < \mu_{s1} = \mu_{s2} = \dots = \mu_{s\tau}$ for up to some threshold number τ of aircraft being monitored, after which the costs might increase at an increasing rate as in (8). Moreover, this structure precludes an explicit consideration of integrality on n_s or binariness on the y -variables, enabling us to equivalently treat these variables as continuous. This penalty structure for workload consideration may be incorporated in the model as follows.

Include the following restrictions within the constraints:

$$\sum_{(i,p) \in C_{sk}} x_{ip} - n_s \leq 0 \quad \forall k = 1, \dots, K_s, s \in S \quad (9a)$$

$$n_s = \sum_{n=1}^{\bar{n}_s} n y_{sn} \quad \forall s \in S \quad (9b)$$

$$\sum_{n=1}^{\bar{n}_s} y_{sn} = 1 \quad \forall s \in S \quad (9c)$$

$$y_{sn} \geq 0 \quad \forall n = 1, \dots, \bar{n}_s, s \in S. \quad (9d)$$

Include the following term within the objective function:

$$\text{(Minimize) } \dots + \sum_{s \in S} \sum_{n=1}^{\bar{n}_s} \mu_{sn} y_{sn} \quad (9e)$$

2.3 Conflict Constraints:

For each sector, let us discretize the horizon into time segments whose durations depend on the amount of traffic normally present in the sector and its conflict resolution capability. This capability is to be reflected within the conflict constraints developed below which impose the restriction that for each sector, the maximum number of (permissible) conflicts that need to be resolved for each time segment should not exceed 1. For example, since air traffic is dense in New York, and an ATC controller handling a New York enroute sector is trained to be more exposed to relatively larger workloads, the duration of the time segments in the corresponding sector could be relatively smaller.

In order to develop these conflict constraints, we must first be able to evaluate each pair of flight trajectories for any potential conflicts. Furthermore, we must be able to determine the sector occupancy of each flight trajectory, and consequently, detect when and over which sectors intrusions leading to collision risk occur. This information is retrieved by running the Airspace Occupancy Model (**AOM**) and the Aircraft Encounter Model (**AEM**) as described in detail by Sherali, et al. (1999). The sector occupancy information is derived from AOM as discussed above. The model AEM then performs a conflict analysis based on the output of AOM.

If any detected conflict is declared to be *fatal* in this analysis by virtue of an aircraft penetrating the inviolable airspace surrounding another aircraft, we would immediately impose a constraint that permits the selection of at most one such flight-plan. Denoting FC as the set of such *fatally conflicting* pairs of flight-plans $P \equiv (i_1, p_1)$ and $Q \equiv (i_2, p_2)$, we begin by stipulating that

$$x_P + x_Q \leq 1 \text{ for all } (P, Q) \in \text{FC}. \quad (10)$$

Other nonfatal conflicting situations are permitted to exist, provided that they can be resolved by the ATC in the particular sector in which they occur. To reflect this resolution capability, we formulate the following set of additional conflict constraints. Suppose that we construct a graph $G_{st}(N_{st}, A_{st})$ for each sector s and time segment t , where N_{st} is the set of nodes that represent all the flight-plans (i, p) which reside in sector s during time segment t , and A_{st} is the set of edges such that if flight-plans P and Q are in conflict in sector s during this time segment t , then A_{st} includes an edge joining these corresponding nodes. If a flight-plan residing in an adjacent sector conflicts with a flight-plan residing within s during the time segment t , then we include such a flight-plan in N_{st} , with the corresponding conflict edge being incorporated within A_{st} . Since we have explicitly excluded non-permissible conflicts via (10) above, we can restrict our attention to recording via A_{st} just the *permissible conflicts*, that is, conflicts that can be resolved by some defined measure. We now impose the constraint that no more than one permissible conflict should occur for each sector during each time segment.

To model these constraints, for each sector, consider the edges in A_{st} taken two at a time, and for each pair k , let S_k be the set of nodes

(representing flight-plans) at which this pair of edges is incident. $|S_k|$ equals three or four, depending on whether the pair of edges is adjacent or not. The imposed constraint would then be

$$\sum_{P \in S_k} x_P \leq |S_k| - 1. \quad (11)$$

Note that there would be $|A_{st}|(|A_{st}| - 1)/2$ inequalities of the type (11) for each sector s , for each time segment t . For notational convenience, let us assume that the index k runs contiguously for $k = 1, \dots, K$ over the constraints (11) for all s, t . Observe that there will likely be several redundant constraints established via this process. We have developed an efficient procedure for filtering these constraints and determining a nonredundant subset of these restrictions. Let this subset of (11) be indexed by $k \in K_{NR}$.

An *Airspace Planning Model*, **APM**, that incorporates the foregoing equity, workload and conflict constraints, along with the suitable costs in the objective function as described above, can now be constructed as a mixed-integer 0-1 programming problem.

To aid us in solving this model, we identify certain special substructures of conflict graphs G_{st} for which facetial cutting planes may be derived in order to obtain a tighter reformulation of the Airspace Planning Model. For some of these structures, we are able to prescribe polynomial descriptions of the corresponding convex hull representations. This analysis leads to the design of some proposed procedures for generating strong cutting planes for enhancing the model representation. Computational results will be reported using data provided by the FAA pertaining to present and future air traffic scenarios, under various concepts of operation.

3 Conclusions

In this paper, we have proposed a 0-1 mixed-integer Airspace Planning Model (APM) to provide guidelines in a collaborative decision-making process between the FAA and commercial airlines. This process seeks to select flight-plans that satisfy various equity, workload, and safety considerations under future airspace scenarios. We have enhanced the solvability of the model by exploring special subgraphs corresponding to the conflict constraints based on which suitable facets of the underlying convex hull representation may be readily obtained. We have also developed accompanying separation procedures for identifying such subgraphs along with the associated valid inequalities they generate, and have prescribed an implementation strategy for solving the resulting tightened problem.

This model can be utilized in one of two ways.

- (a) *Air Traffic Management: Generator of a suitable mix of flight-plans for a set of flights.* In this role, the model can be coordinated with NASPAC, a large-scale simulation model for analyzing airport operations related to a given set of flight-plans, by using the latter simulation package to evaluate in more detail the airport operations related to the prescribed solution suggested by the model.
- (b) *Policy Evaluator.* Various what-if scenarios can be evaluated by policy/decision-makers in determining operational guidelines.

In particular, the following types of investigations can be considered.

- (a) Alternative restrictions on the cordoning of airspace around the RLV spaceport during launches could be evaluated with respect to this model. Different airspace restrictions would yield different values of cost coefficients in the model based on fuel and delay computations. In addition, one might develop certain measures of safety, and incorporate appropriate penalties in the objective cost coefficients to reflect the relative safety of trajectories with respect to RLV operations.
- (b) The effect of various ATC policies can be evaluated with respect to their influence on the parameters \bar{n}_s , along with their associated costs.
- (c) The effect of alternative flight-plans can also be evaluated using this model. In fact, this model can itself serve to evaluate the efficacy of various flight-plan generation programs.
- (d) Similar to (b), different regulations imposed by FAA might yield different interpretations on what poses a “conflict.” These policies could be evaluated by translating them into appropriate constraints of the type (9a-c) and examining their effect on the model solution.

Hence, the model can be used, both in a tactical decision-making mode as an Air Traffic Management tool, as well as for formulating strategic guidelines and policies.

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