# PRICING POLICIES FOR AIR TRAFFIC ASSIGNMENT

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**Abstract:**

The growth of the air traffic demand leads to acute delays and important congestion. In a situation where it is becoming difficult to increase capacity, one way to reduce congestion is to modify the flight plans in order to adapt the demand to the available capacity. This paper shows how a pricing policy could encourage the airline companies to modify the departure times and the routes of their flights. A model of the relation between taxes charged to one aircraft flying a given sector in a given time period and the choice of routes and take-off time by airline companies is proposed. Parameters of the model are tuned by minimizing the difference between an observed situation and the output of the model. Then, the difference, in terms of take-off time and route, between a target assignment and the assignment resulting from taxes is minimized with respect to the prices. Two optimization methods are adapted to the problem and tested on examples. The results show that the optimum can sometimes be obtained by a simple gradient method and that the pricing policy significantly reduces the difference between target and actual assignment.

**Keywords** : Air Traffic Management, Pricing, Gradient, Simulated Annealing.

## 1 Introduction

Since ten years, the air traffic grows in a constant way and the forecasts indicate that this tendency will continue. In Europe as in the United States, the significant growth of the air traffic (approximately 8% per year) is at the origin of the congestion observed not only on large airports but also on space. Indeed, airspace where the density of air traffic becomes significant has to be controlled to ensure the safety of the flights. The airspace under control is divided into sectors. A sector is a volume of the space defined by a floor and a ceiling, and is crossed by air routes. A sector is assigned to a controller which has a global view of the current traffic distribution in the airspace and can give orders to the pilots in order to avoid collisions. If the number of planes, the number of conflicts and the input flow in a sector are too high, the workload of the controller increases and he is not able to ensure his work in optimal conditions of security. A sector in this situation is congested and reactive procedures have to be applied in order to ensure the safety. Congestion can be reduced by modifying the structure of the airspace (increasing the number of runways, increasing the number of sectors by reducing their size).

An alternative way to reduce congestion is to perform flow control. Ground delay programs consist in controlling the flow at its origin. Several previous studies have focused on the slot allocation to reduce congestion. As shown by Maugis (1996), this problem consists in finding departure slots for all flights so that en-route capacities and declared airports capacities are respected and that the total ground delay is minimized. Oussedik and Delahaye (1997) extended this problem including route allocation. In this approach, the modification of flight plans of aircraft, by changing their slot of departure and their route, reduces congestion of sectors. This optimization of new slots and routes for each aircraft is performed with a genetic algorithm and significantly reduces the peak of workload in the most congested sectors and in the most congested airports.

However, the application of flight plans modifications can be a source of problems. Indeed, the allocation optimized for the system doesn't respect equity between users. For instance, for a same Origin-Destination pair, two users can be affected on routes having very different costs. That is why, routes and slots cannot be directly imposed to the companies. In the traffic theory, the difference between a system approach and a user approach has been highlighted by Wardrop (1952). In the system approach, a route and a departure time is assigned to each user by a central organism. In the second approach, users are free to choose their route and their departure time.

The impact of travel time information on users' choices has been modeled and associated traffic assignment techniques have been developed by Cascetta and Cantarella (1991). The evolution of users' choices is modeled by introducing a learning mechanism based on generalized transportation costs which spreads the traffic demand in time and in spatial dimension. Congestion is expected to be reduced by moving the time of departure and by changing the current path. Ben-Akiva, De Palma and Kanaroglou (1986) study the impact of a toll on users' choices in term of route and departure time. They show there exists a toll that brings the congestion to a minimum.

In the problem addressed by this paper, the route-slot allocation which provides the minimum of congestion from the point of view of the system is known. It is supposed to be computed by an existing method such as the one developed by Oussedik and Delahaye (1997). An economical strategy to reach this allocation is searched. The purpose of this paper is to find a mechanism of pricing so that the choices of companies lead to the target allocation.

This paper has been organized as follows. In the next section, a mathematical formulation of the model is given. Two sections are devoted to the formulation of, respectively, parameter identification and price optimization problems. Then, the principle of resolution of the optimization problem is explained. Finally, some numerical examples of traffic are presented.

## 2 Model formulation

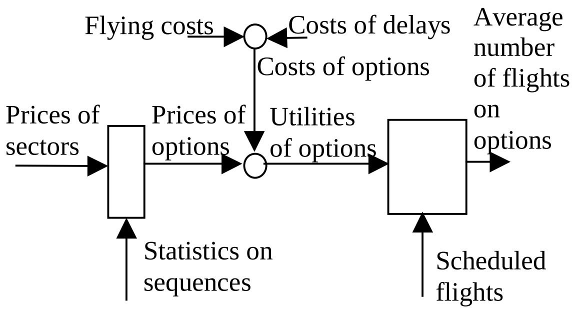
### 2.1 Structure of the model

Figure 1 presents the relation between the different variables of the model. This model is used to estimate the average number of flights expected on each option for given values of sector prices. In order to perform this task, reliable information about scheduled flights, sequences statistics and airline companies costs must be available.

### 2.2 Basic notations and general assumptions

For a given day and for each Origin-Destination pair, it is assumed that the requested time of each flight is known. To fill out flight plans, airline companies have to choose among different options of routes and departure times by taking into account different indicators (delay, travel time). It is assumed that the introduction of a price modifies the choices of companies.

**Figure 1:** Schematic representation of the model




Some of the notations used throughout this paper are given as follows :

| | |
|---|---|
| $W$ | Set of Origin-Destination (OD) pairs |
| $\omega$ | An element of set $W$ |
| $R_\omega$ | Set of alternative routes for OD $\omega$ |
| $T$ | Number of alternative time periods in the horizon |
| $\Delta$ | Length of a time period |
| $S$ | Number of sectors |
| $(i, j)$ | Option of route $i$ and take-off period $j$ |
| $NVP^\omega(j)$ | Number of planned flights on period $j$ |
| $ND^\omega(i,j)$ | Desired number of flights on option $(i, j)$ |
| $NE^\omega(i,j)$ | Average number of flights on option $(i, j)$ |
| $P^\omega_{(i,j)}$ | Price of option $(i, j)$ |
| $x_{k,n}$ | Price of sector $k$ during the period $n$ |
| $X$ | Vector of sector prices for all sectors and all periods |
| $C^\omega_u(i, j)$ | Cost of option $(i, j)$ for a flight initially planned on period $u$. |
| $c^\omega(i, j)$ | Cost of option $(i, j)$ associated to the flight itself. This cost depends mainly on the choice of the route $i$ but may also depend on the departure period $j$ because time varying meteorological conditions can have an impact on aircraft fuel consumption. |
| $r(u, j)$ | Cost of delay for a flight initially planned on period $u$ and taking off on period $j$. This cost covers several aspects such as attractiveness of the airline company for the passenger and problems induced by the perturbation of the planning of aircraft and crew for next flights. |

The cost $C^\omega_u(i, j)$ can be expressed by the summation of two terms; the cost associated to the flight itself and the cost associated to the disrespect of the schedule.

$$C_u^{\omega}(i,j) = c^{\omega}(i,j) + r(u,j)$$

### 2.3 Price for an option

The strategy of pricing could be to consider each origin-destination pair separately. In this case, a price for each option $(i,j)$ and for each OD should be calculated. But with this strategy, flights of different OD which travel through the same sector do not pay the same tax. That is why it is preferred to associate a tariff with each sector and over each period. The price of option $(i,j)$, $i \in R_{\omega}$, is a combination of sector prices :

$$P_{(i,j)}^{\omega} = \sum_{k=1}^{k=S} \sum_{n=1}^{n=T} a_{(i,j)}^{(k,n)} x_{k,n}$$

where $x_{k,n}$ represents the price of sector *k* over period *n* and $a_{(i,j)}^{(k,n)}$ the probability, for an aircraft taking option *(i,j)* , to enter the sector *k* during the period *n.*

This probability is computed taking into account some variability because the sequence of sectors crossed by an aircraft depends not only on the route but also on the type of the aircraft. Indeed aircraft present different rates of climb, and flights levels are chosen in function of aircraft characteristics. It is assumed that for each route a statistic of sequences of sectors and associated entry times are known. For each route, the probability $q_i^l$ is associated to the $l$ th sequence of sectors ( $seq_i^l$ ). For each sector *k* in $seq_i^l$ the interval of time between the take-off and the entry time in the sector ( $t_{i,k}^l$ ) is given.

Let be :

- $b_{(i,j)}^{(k,n,l)}$ the probability that an aircraft on option $(i,j)$ and travelling the $l$ th sequence of sectors enters the sector $k$ during the period $n$
- $E(z)$ integer part of real *z*
- $D(z)$ decimal part of real *z*

then $b_{(i,j)}^{(k,n,l)}$ is given by :

$$b_{(i,j)}^{(k,n,l)} = \begin{cases} 1 - D(\frac{t_{i,k}^l}{\Delta}) & \text{if } k \in seq_i^l \text{ and } n = j + E(\frac{t_{i,k}^l}{\Delta}) \\ D(\frac{t_{i,k}^l}{\Delta}) & \text{if } k \in seq_i^l \text{ and } n = j + E(\frac{t_{i,k}^l}{\Delta}) + 1 \\ 0 & \text{otherwise} \end{cases}$$

Finally, $a_{(i,j)}^{(k,n)}$ is given by the average value over the sequences :

$$a_{(i,j)}^{(k,n)} = \sum_{l} b_{(i,j)}^{(k,n,l)} q_i^l$$

### 2.4 Model of choice behavior

Travel decisions entail choices among discrete set of alternatives, such as departure time, destinations, and routes. The outcomes of such choice are provided by a class of mathematical models called by Horowitz (1983) probabilistic discrete choice models. These models give the probability that a traveler will be attracted to a given alternative among the set of alternatives available to him. The multinomial Logit and multinomial Probit are two well-known examples of probabilistic discrete choice models; see Dial (1971), Yang (1997) and Akamatsu (1997). The Probit model is more complex and the Logit model is particularly applicable to travel demand. Therefore, the Logit model is adopted in this paper to predict the decisions of airline companies for their flights. Their decisions depend on the utility associated to each option. The utility of a trip via route $i$ departing at period $j$ for a flight initially planned on period $u$ is assumed for simplicity to have the linear form :

$$V_u^{\omega}(i,j) = C_u^{w}(i,j) + P_{(i,j)}^{\omega}$$

The probability that a flight planned on slot $u$ is assigned to option $(i,j)$ is expressed as follows :

$$PR_u^{\omega}(i,j) = \frac{\exp\left[-\alpha V_u^{\omega}(i,j)\right]}{\sum_{r \in R_{\omega}} \sum_{s=u-1}^{s=u+2} \exp\left[-\alpha V_u^{\omega}(r,s)\right]}$$

In this formula, the assumption that the moving of the slot of departure must be done in a limited domain ($[J_{\min}, J_{\max}]$) is made. These predictions give the average number of flights assigned to option $(i,j)$ :

$$NE^{\omega}(i,j) = \sum_{u=j-J_{\max}}^{u=j+J_{\min}} NVP^{\omega}(u) PR_u^{\omega}(i,j)$$

with $NVP^{\omega}(u) = 0$ for u<1 and u>T.

## 3 Identification Problem

Assuming that for each period $j$ and each route $i$ the number of flight taking off, $NO^{\omega}(i,j)$, has been observed, some parameters of the model can be estimated. The problem consists in minimizing the square differences between the observed number of flights and the average number of flights given by the model:

$$\min \sum_{\omega \in W} \sum_{i \in R_\omega} \sum_{j=1}^{j=T} \left[ NO^\omega(i,j) - NE^\omega(i,j) \right]^2$$

The costs perceived by the companies are usually not well known. For that reason the minimization has to be performed with respect to the variables related to costs: $c^\omega(i,j)$ and $r(u,j)$. Identification of the $\alpha$ parameter of the Logit model is not relevant. Indeed, a modification of this parameter is equivalent to a change of unit for the expression of the costs and prices. In order to have a ratio number of measurements / number of identified parameters high enough it may be necessary to make additional assumptions on the cost structure. Two reasonable assumptions are:

The meteorological conditions prediction is so uncertain that the predictable part of the cost associated to the flight itself depends mainly on the route:

$$c^\omega(i,j) = c^\omega(i)$$

The cost induced by a delay is proportional to the absolute value of the deviation with respect to the schedule:

$$r(u,j) = \beta |j - u|$$

With such assumptions, the error criterion has to be minimized with respect to $\beta$ and with respect to the different $c^\omega(i)$.

## 4 Optimization Problem

It is assumed that an ‘‘optimal” route and slot allocation has been found. The problem consists in finding prices that minimize the difference between the average number of flights on each option of each OD couple and the desired number of flights.

$$\min_X \sum_{\omega \in W} \sum_{i \in R_\omega} \sum_{j=1}^{j=T} \left[ ND^\omega(i,j) - NE^\omega(i,j) \right]^2$$

The average number of flights is the result of the choices of companies. The desired number of flights on each option is the result of a slot and route allocation which can be obtained by the method of Oussedik and Delahaye (1997).

The sector prices are positive variables and should be small with respect to the costs of options. In order to compute only realistic prices the following constraint should be used:

$$0 \le x_{k,n} \le \bar{x} \text{ for } k = 1,S \text{ and } n = 1,T$$

where $\bar{x}$ is the maximum allowable price for a sector.

The similarity between the identification problem and the price optimization problem allows the use of the same kind of optimization methods for the resolution of both problems.

## 5 Principle of resolution

As the criterion of the price optimization problem is sometimes non-convex, it is interesting to assess the resolution of the problem by local and global optimization methods. A gradient algorithm and a simulated annealing algorithm are proposed for obtaining respectively local and global minima.

### 5.1 Gradient Algorithm

The gradient algorithm modifies the prices $x_{k,n}$ iteratively; see for instance Minoux (1990). At each iteration $q$ the new prices $x_{k,n}^{q+1}$ are computed from the current prices $x_{k,n}^q$ by performing the following steps :

- Computation of the sensitivity $\nabla f_{k,n}^q$ of the criterion with respect to each price by : $\nabla f_{k,n}^q = \left. \frac{\partial F(X)}{\partial x_{k,n}} \right|_{X = X^q}$
- Optimization in the direction defined by $\nabla F(X)$ taking into account the constraints : $x_{k,n}^{q+1} = \min\left( \max(x_{k,n}^q - \lambda^q \nabla f_{k,n}^q, 0), \bar{x} \right)$
- At each step $F(X^{q+1}) \le F(X^q)$

The algorithm stops either when the criterion reaches a given value or when the number of iterations is too large.
At each iteration $\lambda^q$ is obtained by minimizing the criterion with respect to this parameter using a golden search method that requires 10 computations of the criterion.
An analytical expression is used for $\nabla f_{k,n}^q$ :

$$-2 \sum_{\omega} \sum_{i \in R_\omega} \sum_{j=1}^{j=T} \left[ ND^\omega(i,j) - NE^\omega(i,j)[X] \right] \nabla nve_{k,n}^\omega(i,j)$$

with :

$$\nabla nve_{k,n}^\omega(i,j) = \sum_{u=j-J_{\max}}^{u=j+J_{\min}} NVP^\omega(u) PR_u^\omega(i,j) \nabla pr_{k,n}^{\omega,u}(i,j)$$

and :

$$\nabla pr_{k,n}^{\omega,u}(i,j) = -\alpha \left[ a_{(i,j)}^{(k,n)} + \frac{\sum_{r \in R_\omega} \sum_{s=u-J_{\min}}^{s=u+J_{\max}} a_{(r,s)}^{(k,n)} \exp\left[-\alpha V_u^\omega(r,s)\right]}{\sum_{r \in R_\omega} \sum_{s=u-J_{\min}}^{s=u+J_{\max}} \exp\left[-\alpha V_u^\omega(r,s)\right]} \right]$$

### 5.2 Simulated annealing algorithm

The simulated annealing algorithm works iteratively with two values for the prices :

- $\tilde{X}^q$ the best value obtained until iteration $q$
  $X^q$ the value for iteration $q$
- At each iteration, the steps of the algorithm are
- Sort of a price $Y^q$ in the neighborhood of $X^q$
  If $F(X^q) < F(Y^q)$
  - $X^{q+1} = Y^q$
  - If $F(Y^q) < F(\tilde{X}^q)$
    $\tilde{X}^{q+1} = Y^q$
    else
    $\tilde{X}^{q+1} = \tilde{X}^q$

  Else

  - $\tilde{X}^{q+1} = \tilde{X}^q$
  - If $random[0,1] < \exp\left(F(X^q) - F(Y^q) / T^q\right)$
    $X^{q+1} = Y^q$

    else
    $X^{q+1} = X^q$

The algorithms stops either when the criterion reaches a given value or when the number of iterations is too large. Three versions of the algorithm exist. Those versions differ in the neighborhood definition. For the first version, each component $y_{k,n}^q$ of $Y^q$ is sorted in an uniform distribution over the interval $\left[x_{k,n}^q - \delta, x_{k,n}^q + \delta\right]$ The two other versions of the algorithm use a sector load factor given by :

$$L_{k,n}[X] = \sum_{\omega} \sum_{i \in R_\omega} \sum_{j=1}^{j=T} a_{(i,j)}^{(k,n)} \left[ND^\omega(i,j) - NE^\omega(i,j)[X]\right]$$

For the second version, if $L_{k,n}[X] > 0$, $y_{k,n}^q$ is sorted in an uniform distribution over the interval: $\left[x_{k,n}^q - \frac{\delta}{2}, x_{k,n}^q + \frac{3\delta}{2}\right]$. Otherwise it is sorted in: $\left[x_{k,n}^q - \frac{3\delta}{2}, x_{k,n}^q + \frac{\delta}{2}\right]$. For the third version $y_{k,n}^q = x_{k,n}^q + random[0,\delta]L_{k,n}\left[X^q\right]$. For all versions, when the sorted value is outside the interval $[0, \bar{x}]$, it is projected on one of the interval boundaries. The variable $T^q$ decreases by steps as $q$ increases:

$$T^{q+1} = \begin{cases} \lambda T^q \text{ if } q \equiv 0[m] \\ T^q \text{ otherwise} \end{cases}$$

where $0 < \lambda < 1$, $m$ is the step length and $T^1$ is given.

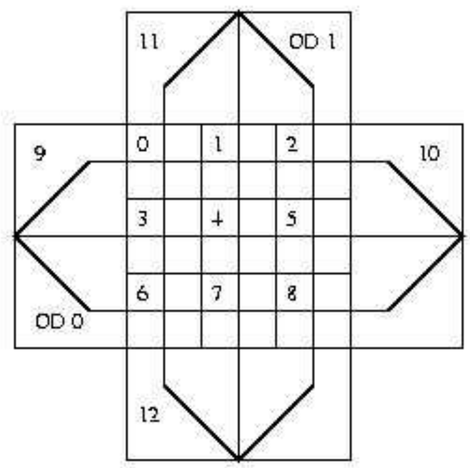
## 6 Numerical experiment

### 6.1 An academic example

6.1.1 Scenario

The network used is presented on figure 2. It includes two OD couples and three routes for each OD. The control is performed through 13 sectors. The capacity of sectors 0 to 8 is assumed to be one aircraft per sample time and the capacity of sectors 9 to 12 is assumed to be three aircraft per sample time. For each route there is only one sequence of sectors and the time to travel any sector is equal to one sample time.

**Figure 2:** Network for the academic example




The time horizon considered is equal to eight sample times. The demand for each OD consists in three aircraft scheduled at the first sample time. The maximum ground delay that airlines integrate in their choice is equal to three sampling periods. The $\alpha$ parameter is set to 0.1.

6.1.2 Identification

It is assumed that in the observed situation each aircraft takes off on time and flies on the shortest path (sector sequences {9, 3, 4, 5, 10} for OD 0 and {11, 1, 4, 7, 12} for OD 1). This situation induces large overload of sectors culminating with a load of 6 aircraft per sample time in sector 4 at the third sample time.

Some remarks about this identification problem must be done:

⇒ Some possible options are not used. In theory, it corresponds to those options infinite costs. In order to obtain finite costs, the observed value for unused options is set to 0.1 and consequently the number of aircraft taking off on time on shortest path is set to 2.89 instead of 3.

⇒ If, for an OD, a constant is added to the utility of all options the result of the formula giving the probability of an aircraft choosing an option is not changed. Thus, it is not possible to identify directly costs, but only the difference between the costs of different options.

⇒ The problem presents symmetries ; there are no differences between unused routes. Demands and observed situations are equal for the two ODs.

As a consequence, the identification problem consists in finding only two parameters: $\beta$ and the cost difference between the unused routes and the shortest path routes.

This identification was performed with a simulated annealing algorithm and leads to a criterion of 0.007102. The optimal values are $\beta = 45$ and a cost difference of 46.3. The computation of the cost of the shortest path routes is performed assuming that the cost of flying is equal to three times the cost of a ground delay. Considering that the flying time is 5 sample periods, this leads to a value of 675.

6.1.3 Optimization of prices

The target was constructed manually in order to respect the sector capacity constraints. For OD 0 aircraft must be on time and each aircraft must take one of the three routes. For OD 1 one aircraft must be delayed for one period and take route {11,0,3,6,12}, another aircraft must be delayed for two periods and take route {11,1,4,7,12} and the last aircraft must be delayed for three periods and take route {11,2,5,8,12}.

The bound on sector prices was computed assuming that the impact on total company cost should remain reasonable : As each route crosses five sectors the cost of the shortest path route is divided per five and the maximum price of a sector is obtained by taking a proportion (between 0.05 and 0.95) of the result. The value obtained for $\bar{x}$ varies between 6.75 and 128.25.

Prices were optimized using the gradient algorithm and the third simulated annealing algorithm. The gradient algorithm does not reach the global minimum when the maximum price is high. Figure 3 presents the results obtained with the simulated annealing.

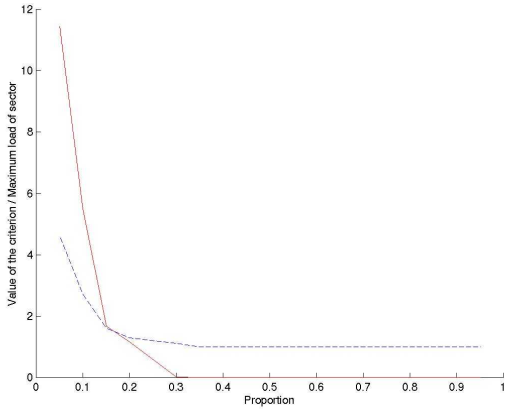
Those results indicate that:

- High prices (about 35% of the cost of the shortest path route) are needed to reach the target situation.

- For smaller prices the target situation is not reached, but the increase of the price decreases continuously the load of the sector with highest flow.

This academic example show that a pricing policy could be an adequate tool in the treatment of a saturation reduction problem.

**Figure 3:** Relation between the bound on sector prices (given as a proportion of the cost of the shortest route) and the optimization results – Optimization criterion ; distance to target situation (solid line) – Largest sector load for sectors 0 to 8 (dashed line)




**6.2 Optimization for more realistic example**

6.2.1 Scenario

- Network

The algorithms previously described are tested on a part of the French airspace which includes twenty-three sectors. Four origin-destination pairs are considered. One pair presents four routes and the others three routes.

- Demand

The demand corresponds to the traffic measured on these OD with a sample time of fifteen minutes. The $\alpha$ parameter of the Logit model is set to 0.115. Two demands are used; their optimization horizon are respectively the morning peak (5h00-8h00) and the all day (00h00-24h00). The constraints on maximum prices are not activated.

- Target

For both optimization horizons, two target traffic assignments are used. For the morning peak, the first target corresponds to an uniform distribution over time periods and routes, and the second one to integer numbers of flights randomly allocated on routes and periods. For the all day, the first target corresponds to an uniform distribution over the routes, and the second one to an uniform distribution over time periods and routes.

- Size of the problem

For the morning peak and the all day, the number of optimization variables $S \times T$ are respectively $23 \times 12 = 276$ and $23 \times 96 = 2208$

4.2.2 Results

- Elementary computation time

On a SUN-ULTRA 10 with 64 Mo Memory, time for computing one value of the criterion, one value of the gradient and one value of the load factor for the morning peak are respectively $T_c$=0.02, $T_g$=0.54 and $T_l$=0.03. For the all day demand, the values are $T_c$=0.06, $T_g$=10.6 and $T_l$=0.36. The computation time associated to tests and random generation are negligible. The gradient algorithm performs one gradient computation and about ten criterion evaluations per iteration, while the simulated annealing performs only one criterion evaluation per iteration. Thus a fair comparison of the algorithms implies a number of iterations $\frac{T_g + 10T_c}{T_c}$ higher for the first version of the simulated annealing than for the gradient. The values of this ratio are respectively 37 and 186 for the morning peak and the all day demand. For the two others versions of the simulated annealing the number of iteration should be $\frac{T_g + 10T_c}{T_c + T_l}$ higher than for the gradient. The values of this ratio are respectively 14.8 and 26 for the morning peak and the all day demand.

- Gradient

Figure 4 and 5 depict the evolution of the criterion during the optimization for the gradient algorithm.

The distance to the target decreases significantly in the 100 first iterations and further criterion improvement is obtained continuously. Table 1 presents the values of the criterion before and after optimization.

**Figure 4:** Evolution of the criterion - Gradient algorithm - All day second target (solid line) - Morning peak first target (dashed line)

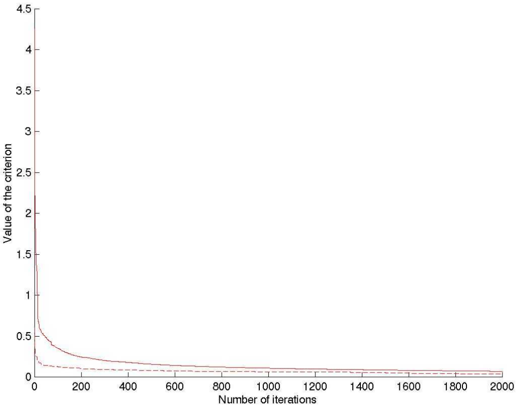



**Figure 5:** Evolution of the criterion - Gradient algorithm - All day first target (solid line) - Morning peak second target (dashed line)

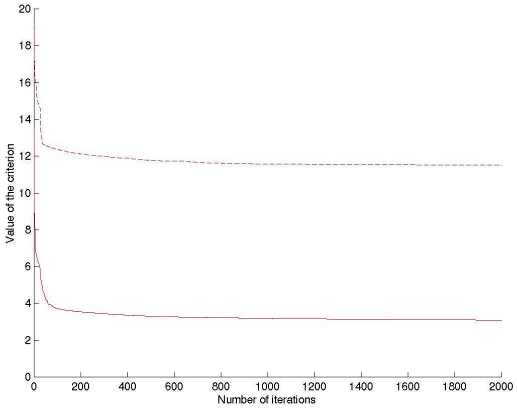



**Table 1:** Criterion after 2000 iterations (nb of flights)[2] for an optimization with the gradient algorithm

| Demand | Morning Peak | | All day | |
|---|---|---|---|---|
| Target | 1 | 2 | 1 | 2 |
| Null prices | 0.62 | 19 | 13 | 4.3 |
| Result | 0.036 | 12 | 3.1 | 0.069 |

- Simulated annealing

The criterion given by the second and third version of the simulated annealing for the second target of the morning peak are between 14 and 16 for $\delta$ and $T^1$ varying on [0.1-10] and [0-0.5] respectively. For the same problem, the first version of the algorithm gives value between 12 and 14. For this version, values of $\delta$ =0.1 and $T^1$=0.02, allow good solutions for the morning peaks' problem : the criteria obtained after 74000 iterations are respectively between 0.072 and 0.095 (depending of the initial value of the random generator) for the first target and 12 for the second target.

**Figure 6:** Evolution of the criterion - Simulated annealing algorithm - Morning peak first target (two different initial values of the random generator)

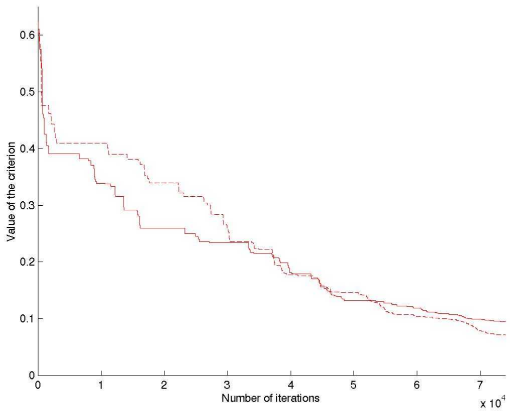
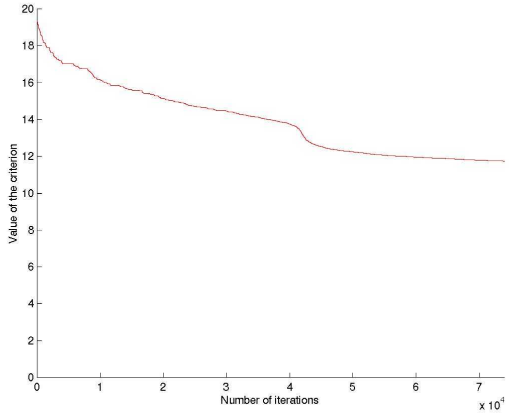



Figure 6 presents the evolution of the criterion for the first target and two different initial values of the random generator.

The evolution of the distance to the second target during the optimization with the simulated annealing algorithm is shown on figure 7.

**Figure 7:** Evolution of the criterion - Simulated annealing algorithm - Morning peak second target




The criterion decreases from 19 to 12 in 74000 iterations.

- Comments

The results obtained lead to the following comments :

⇒ The optimization by the two algorithms shows that a large decrease of the initial value of the distance to the target is obtained. This indicates the feasibility of the flow control by prices.

⇒ The gradient algorithm is more efficient than the simulated annealing algorithm : for the first target and a similar computation time the simulated annealing algorithm finds a solution with a value between 0.072 and 0.095 while the gradient produces a solution with a criterion of 0.036. Moreover, the decrease of the criterion in the first iterations is significantly higher for the gradient. It seems that for this problem, there is no local minimum that could give an advantage to the simulated annealing.

⇒ The convergence of the gradient is very slow : the criterion is significantly reduced in the first iterations but further improvements are obtained asymptotically.

⇒ For the gradient method, a significant number of prices are equal to zero, indicating that prices are only raised locally to avoid congestion. This could be the reason of the convergence to a local minimum in the academic example.

## 7 Conclusion and future work

A simple assignment model of the relationship between the toll applied to aircraft flying a given sector at a given instant and the choice made by airline companies in terms of take-off time and route has been proposed. For a given demand, the model is able to provide the flow at the origin for each route all along the day. An identification procedure for estimation of some parameters of the model has been proposed and demonstrated on an academic example. An optimization problem which corresponds to the objective of setting those flows at target values has been formulated. The solution of the problem by two kind of algorithms, gradient and simulated annealing, has been studied. The results obtained by the two methods on actual data are similar indicating that the criterion does not present local optima. However, for the academic example with high bounds on prices, the gradient algorithm converges to a solution that is significantly worst than the solution of the simulated annealing. This indicates the presence of local minimum. The conditions for the existence of a local minimum remains to be clarified. The target values are not necessary reached indicating that the system optimum may not always be obtained trough a sector pricing policy.

Research on pricing policies for the reduction of congestion in the area of air transportation is a new domain and many options can be studied. Concerning the specific approach proposed in this paper, further research will address direct optimization of congestion and modeling of the traffic which does not follow timetables.

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