Airspace Fractal Dimensions and Applications

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Abstract

The fractal dimension is presented as a metric for the evaluation of alternative traffic patterns emerging from differing operational concepts. It is shown through analysis and related example, that the number of possible conflicts per hour (for each aircraft) grows linearly with the number of flights. The slope of this curve is then shown to vary as the power of the fractal dimension. A framework, based upon gas dynamics, is presented to explain this relationship. The fractal dimension for all flights across the NAS is computed for multiple operational scenarios from current operations, to free-flight with RVSM and full cruise-climb. Nationally, the fractal dimension is shown to increase as more freedom is provided to operators. In transition areas conversely, the free-flight scenarios show a decrease in fractal dimension, along with a corresponding increase in the conflict rate. In addition to providing information on the scalability of conflicts, the fractal dimension also provides information on the number of degrees-of-freedom in the traffic flow. It is suggested that as the number of degrees-of-freedom increase, the complexity of the traffic may also increase. The fractal dimension for various ARTCCs under current operations is presented, illustrating variations in the number of degrees-of-freedom in the different ARTCCs.

Introduction

As future operational concepts are presented, metrics are required to evaluate the anticipated performance of the airspace operating under these alternative operational concepts. Prior research has presented several methods for evaluating airspace complexity and dynamic density (e.g.,[2]-[5]). Several of these methods are dependent on both the traffic and airspace sectorization. We present an alternative approach for computing one element of traffic complexity by calculating the fractal dimension of the traffic pattern. This approach decouples the complexity due to airspace partitioning from that due to traffic flows.

We are all familiar with the dimension of ordinary geometrical entities. A cube has dimension 3, a line dimension one, and so on. The fractal dimension is merely an extension of this concept to more complex geometrical entities that may have non-integer dimensions. A common example of a geometrical entity suggesting a non-integer fractal dimension is the coastline. If one were to measure the coastline, the answer would largely depend on the size of the ruler one was to use. In the limit as we measure the coastline with an infinitesimal ruler, the coastline length would grow ad infinitum.

In order to address the scale dependence, Mandelbrot introduced the notion of fractals, which are simply *self-similar* geometries. The literature contains many examples of fractals and how fractals mimic reality (e.g. rock surfaces, ferns, etc.). One of the interesting aspects of fractal geometries, is that one can often compute a non-integer fractal dimension. The fractal geometry is often defined as the ratio of the log of the number of similar copies to the log of the scale. For example doubling a square (scale = 2) will give four copies of the square. This gives (log(4)/log(2)) a fractal dimension of 2 for a square.

The block count approach is a simpler computational procedure for computing the fractal dimension of a geometrical entity. In this approach, the entire volume is subdivided into a collection of blocks with linear dimension d as shown in Figure 1. A geometrical entity is described in the area, and the number of blocks contained in this entity is counted (say N). The fractal dimension (D_0) can be estimated for this shape through the following.

$$D_0 = \lim_{d \to 0} \frac{\log(N)}{\log(d)}$$

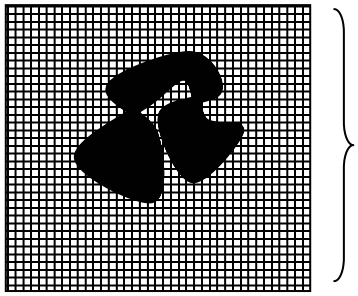

Figure 1. n^2 squares with linear dimension d = l/n

The reader is invited to verify that a simple line will yield a dimension 1, a rectangle 2, and a cube 3.

Application to Airspace Analysis

While this description of fractal dimension may be interesting, the application to analysis of airspace may not be immediately apparent. However, if we consider the migration of the current structured routing system towards a free-flight environment, we begin to understand that the fractal dimension may provide a single number with which to characterize the ensuing route structure.

In the case of the airspace route structure, we can compute the fractal dimension using the block count method as well. The geometrical entity that we are considering is the routes (with altitude information) of all aircraft in the National Airspace System. Figure 2 illustrates an example geometry (not all flights have been included, figure for illustrative purposes only).

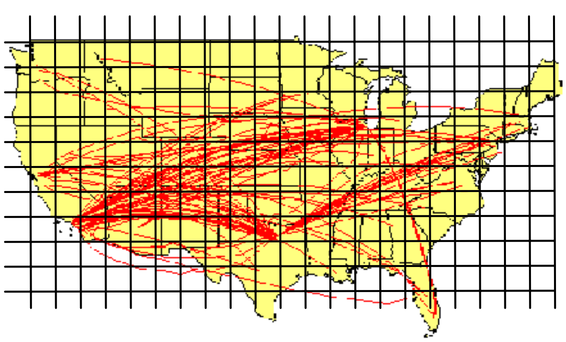

Figure 2. Computation of Fractal Dimension for a route structure.

Under current operations, aircraft cruise on specified flight levels according to the direction of flight, and follow an existing network of linear routes. One can intuit that the dimension of this network of routes is one (linear routes on specified altitudes). As the national airspace system migrates towards free-flight, flights will no longer be confined to these linear routes, and may be allowed to cruise climb. If all of the airspace was covered by routes, the fractal dimension of the future route structure would be 3. However, flights will still be operating between city pairs, and they will tend to follow similar routes according to the best winds, and they cruise at roughly constant altitude. Thus, in practice one would expect the dimension of the route structure to be less than 3.

Fractal Dimension examples

In order to illustrate the fractal dimension of different routes, we computed the fractal dimension, using the block count method, of various examples. In all cases, the fractal dimension could also be obtained analytically. The five cases investigated are described below:

- Flights confined to one route on a single altitude (dimension = 1)
- Flights confined to a single altitude, begin on a line (dimension = 2)
- Flights randomly assigned an altitude, begin on a line (dimension = 3)
- Flights confined to a single altitude, begin on a Cantor set (dimension = 1.63)
- Flights randomly assigned an altitude, begin on a Cantor set (dimension = 2.63)

These cases are shown in Figure 3, and 4.

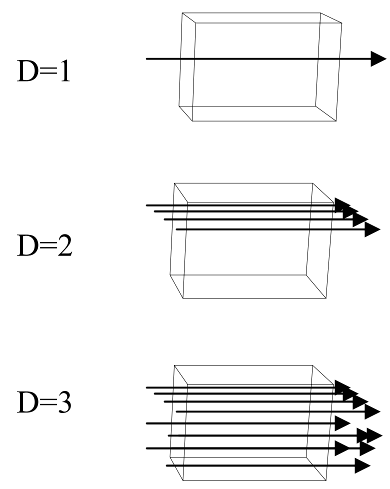

Figure 3. Example route cases investigated.

The Cantor set is an initial condition for the flights into the volume. Rather than being allowed to begin on an axis, these flights are confined to beginning on a subset of the axis defined by a Cantor set. This fractal is derived by recursively removing the middle third of all remaining lines. These examples were included to obtain situations with a known fractal dimension to test the computation procedure.

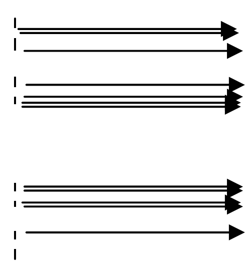

Figure 4. Top view of routes using a Cantor Set as initial condition.

All routes were investigated in a square from 0 to 100 nautical miles, for 1 hour, and were allowed to have ground speeds from 350 to 400 knots. When altitudes could vary, they were allowed to vary uniformly from 28000 to 38000 feet.

Impact of Fractal Dimension on Conflicts

For each of the above cases, an average conflict rate can be computed for each aircraft (conflicts per hour). This conflict rate can also be computed as the number of flights (n_flights) increases. Since the ground speed is varying in all cases, the conflict rate will be proportional to the number of aircraft. Figure 5 illustrates the growth in this conflict rate for the case with fractal dimension = 2. We can see that the conflict rate is linear with the number of flights. Some variation occurs due to the use of random trials to obtain the curve. This linear pattern is repeated for all of the examples in the preceding section. For each of the curves, the slope of the curve can be computed. Conflicts are broken down into two types of conflicts: all conflicts, and those that do not occur at t=0 (occurring at the start of a simulation). Table 1 illustrates the slope of all cases.

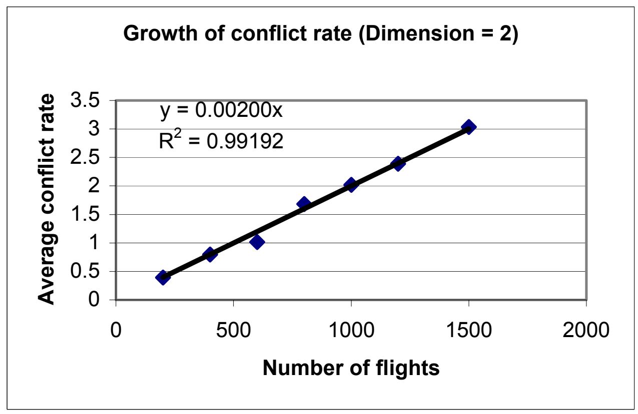

Figure 5. Growth of conflicts with number of flights for fractal dimension = 2

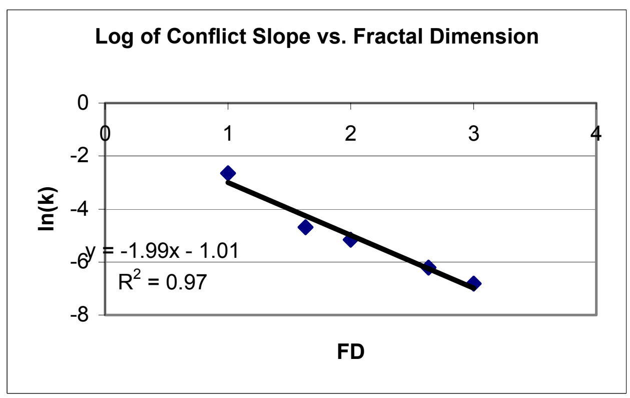

Figure 6. Relationship between slope of conflict rate and fractal dimension.

Table 1. Slope of conflict rate as fractal dimension varies.

Dimension	Slope of conflict rate (all)	Slope of conflict rate (excl. t=0)
1	0.0706	0.0209
1.63	0.0092	0.0030
2	0.0058	0.0020
2.63	0.0020	0.0007
3	0.0011	0.00039

We can further plot the log of the slope of the conflict rate versus the fractal dimension and get a roughly linear relationship as shown in Figure 6.

The linear relationship implied between the fractal dimension and the log of the conflict rate slope can be derived theoretically through analogy with gas dynamics (e.g., [6]). Suppose we have N aircraft on a manifold with linear dimension *l*. The manifold has a fractal dimension of D. We can define a sort of density of aircraft per unit "volume" on this manifold is as follows:

$$\rho = \frac{N}{l^D}$$

We have assumed that aircraft can only be present on the manifold. Thus, on a two-dimensional surface, the density would be expressed as flights per unit area, whereas in a three dimensional volume, the density would be expressed as flights per unit volume. We further assume that each flight will travel with an average *relative* speed c, and require a lateral separation x_{sep}. Each flight will expose an area as follows to other flights:

$$A = x_{sep}^{D-1}$$

Per unit time, a single flight will sweep a "volume" as follows:

$$c x_{sep}^{D-1}$$

Assuming random motion, sweeping the above volume, a flight will encounter a number of flights equal to density multiplied by volume. The number of encounters per unit time, for one flight, can then be expressed as follows:

$$\frac{N}{l^D} c x_{SEP}^{D-1}$$

For each flight, the number of encounters will scale as the number of other flights. However, the slope of this linear relationship can be expressed as:

$$k = \frac{c}{x_{SEP}} \left(\frac{x_{SEP}}{l} \right)^D$$

with a corresponding logarithmic relationship:

$$\ln(k) = D \ln\left(\frac{x_{SEP}}{l} \right) + \ln\left(\frac{c}{x_{SEP}} \right)$$

The implication of this relationship, is that as the fractal dimension is increased, the conflict occurrence will decrease *exponentially* with the fractal dimension. (The reader should note that the length scales should be greater than the separation standard).

Dimension of Free-Flight versus Existing Routes

The box-count method of approximating fractal dimension was applied to the following national-level scenarios. (Obtained using the method described in [7]).

- Routes under current operations. Flights climb to their cruise level.
- Wind-optimized trajectories. Flights are confined to cardinal altitudes based upon current rules for direction of flight. Flights may step-climb.
- Wind-optimized trajectories – RVSM. Flights are confined to altitudes based upon modified rules for direction of flight (1000' separation above FL290).
- Wind-optimized trajectories – Cruise Climb. Flights are allowed to freely choose and modify altitudes along the route.

Current operations were based on filed flight plans. All wind-optimized trajectories were computed by minimizing the fuel cost in order to meet the same time of flight as filed.

The fractal dimension was computed using just the two-dimensional track, and using the full three-dimensional profile. In all cases, we were interested only in that portion of flights over the CONUS and above FL240. The

CONUS was defined as a box with latitude from 20 to 50 degrees North, and longitude from 70 to 125 degrees West. We note that movement towards free-flight yields an increase in the dimensionality of the flights. Note also that current flights are not confined to a totally one-dimensional route structure.

Scenario	Using Track	Track & altitude
Current	1.38	1.31
Wind – Cardinal	1.73	1.49
Wind – RVSM	1.74	1.53
Wind – Cruise Climb	1.72	1.52

Using the results of the conflict analysis, one might conclude that the above increase in the fractal dimension as we transition to free-flight would lead to a reduction in the expected number of conflicts. However, prior research in this area suggests that the total number of conflicts remains constant [8-10], albeit changing in distribution. Specifically, conflicts increase in transition sectors and decrease in en-route sectors.

The table below illustrates the change in fractal dimension as we migrate from current operations to free-flight in a 120 nmi box (from 10,000 feet to FL410) surrounding major airport areas. In all areas, as we migrate to free-flight the fractal dimension *decreases*. We can explain this phenomenon if we consider that arrivals and departures are currently segregated during transition.

Airport	FD Current	FD Wind Cardinal
DFW	1.26	1.17
LAX	1.39	1.29
DEN	1.26	1.15
MIA	1.30	1.23
SFO	1.29	1.26
BOS	1.27	1.23
JFK	1.38	1.28
ORD	1.24	1.18
ATL	1.22	1.17

The above decrease in fractal dimension in major transition areas suggests that the number of conflicts in the above areas should increase under free-flight. Figure 7 shows that this is indeed the case. In agreement with our simple model, a trend is present whereby larger decreases in fractal dimension correlate with greater increases in the number of conflicts.

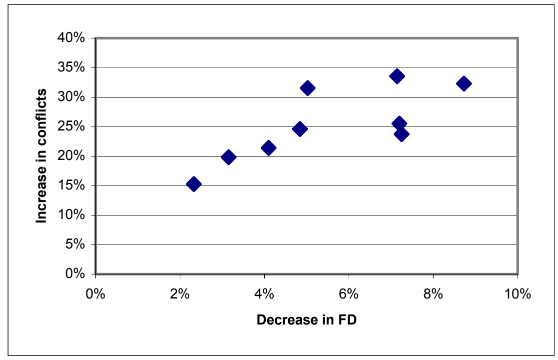

Figure 7. Conflict change versus change in Fractal Dimension.

As defined in the current examples, the free-flight scenarios increase dimensionality en-route, but decrease dimensionality in transition, this leads to the situation in which the total number of conflicts are approximately constant, but conflicts increase in transition and decrease en route.

Degrees of Freedom

In addition to providing insight about the behavior of conflicts as the number of flights scales, the fractal dimension of the airspace provides the analyst with information about the aggregate degrees-of-freedom present in a collection of traffic being analyzed. Examples with known degrees-of-freedom are presented below.

Scenario	Fractal Dimension
Constant Speed Linear network	1.0
Linear Network Random Speed	2.0
Random Heading constant speed	2.0
Random Heading and speed	2.6
Random Heading, Fixed TOD	2.0
Random Heading and random TOD	2.4
Fixed Heading random TOD and descent rater	2.1

Clearly as the number of degrees of freedom (dof) present in the traffic flow increase, the fractal dimension increases to reflect the number of those degrees of freedom present in the traffic pattern. The fractal dimension therefore suggests itself as a possible measure useful in the determination of traffic complexity. The term traffic complexity refers specifically to the traffic pattern. Note that the fractal dimension is *independent* of sectorization and does not scale with traffic volume. Thus, the fractal dimension provides a measure of the geometrical complexity of the traffic pattern, answering the question, "how many degrees-of-freedom are typically changing in this traffic?"

In order to address this dof issue, we computed the fractal dimension of various sections of the country. In transition areas from 10,000 ft to FL240, the fractal dimension was 1.2 for all terminal areas investigated except for NYC which was 1.3.

We also approximated several ARTCCs as shown in the table below. Looking at the ranking of the centers confirms what we know about the traffic within each center.

Center	Fractal Dimension (Above FL240)
ZNY	1.32
ZID	1.28
ZOB	1.28
ZAU	1.26
ZAB	1.24
ZMP	1.16
ZLC	1.13

Conclusions

The above analysis indicates that the fractal dimension may be exploited as a metric for evaluating differences in traffic flow between alternative scenarios. A higher fractal dimension indicates preferable scalability in terms of the number of conflicts. When comparing scenarios, different domains, such as transition sectors, should be considered separately since changes in fractal dimension may not be uniform across domains. A higher fractal dimension also indicates more degrees-of-freedom being used in the airspace. However, a higher fractal dimension may also indicate a higher traffic complexity measure.

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Biographies

Dr. Stephane Mondoloni is chief scientist at CSSI Inc. in Washington, DC. For the past eight years, he has been developing simulation models and conducting analyses for the Federal Aviation Administration and the National Aeronautics and Space Administration. Recent tasks have focused on conflict detection and resolution, aircraft trajectory optimization, and evaluation of alternative operational concepts. He received his Ph.D. in 1993 from MIT in Aeronautics and Astronautics.

Diana Liang works for the Office of System Architecture and Investment Analysis for the Architecture and System Engineering Division. She is responsible for the development of the NAS Architecture Tool and Interface called CATS-I, directing

analyses in support of NAS Concept Validation, and the development of Modeling Tools and Fast-Time Simulations to support that validation. This work includes several models she is developing jointly with NASA and cooperative efforts with Europe via Eurocontrol. Prior to working for ASD, Ms. Liang worked in the Office of Energy and Environment for two years as the lead for the Emissions and Dispersion Modeling System (EDMS), updated the FAA's Air Quality Handbook and reviewed Environmental Impact Statements related to emissions. Ms. Liang holds a BS in Computer Science and is currently attending George Washington University.