TARGET MISS DISTANCE TO ACHIEVE A REQUIRED PROBABILITY OF CONFLICT

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1. Abstract

New operational concepts and tools sometimes propose the resolution of potential conflicts at greater times to closest approach than in current practice. When resolving crossing conflicts horizontally, by changing the relative velocity (that is, by changing the speed or heading of one or both aircraft), nominal minimum distances must allow for errors in the predicted positions of aircraft in order to achieve a sufficiently low probability of conflict. A simple relationship is derived between nominal miss distance, required probability of conflict and the magnitude of the error in the predicted positions of the aircraft involved. Possible trade-offs between nominal miss distance, manoeuvre initiation time and trajectory prediction accuracy are illustrated.

2. Introduction

New operational concepts and tools sometimes propose the resolution of potential conflicts at greater times to closest approach than in current practice. Tools currently under development include MITRE's Problem Analysis and Resolution Ranking (PARR) [Kirk et al.] and EUROCONTROL's Conflict Resolution Adviser (CORA).

A conflict occurs when the altitudes of two aircraft differ by less than the required vertical separation and the distance between them is less than the required horizontal separation. By estimating the future positions of aircraft, potential conflicts can be detected.

A potential conflict is resolved by taking steps to increase the vertical or horizontal distance between the aircraft when they will be closest. However, positions cannot be predicted perfectly and so minimum distances cannot be known with certainty in advance. The action of a flight management system to keep an aircraft on track at a given level to within small tolerances can limit cross-track and vertical errors in predicted positions, but along-track errors grow in an uncontrolled way. When resolving crossing conflicts horizontally by changing the relative velocity (that is, by changing the speed or heading of one or both aircraft), nominal minimum distances must allow for errors in the predicted positions of aircraft in order to achieve a sufficiently low probability of conflict. Assuming that potential conflicts are always resolved, the probability of conflict is equal to the probability that further intervention will be needed.

Previous work [Paielli and Erzberger 1997, Irvine 2002] describes how to estimate probability of conflict. Conflict resolution can be thought of as action to reduce probability of conflict, ideally to a very low level. In this paper a simple relationship is derived between nominal miss distance, required probability of conflict and the magnitude of the error in the predicted positions of the aircraft involved. Possible trade-offs between nominal miss distance, manoeuvre initiation time and trajectory prediction accuracy are illustrated.

3. Conflict probability estimation

This section summarises the approach to conflict probability estimation given in [Irvine, 2002] and includes the following steps:

- Calculation of the minimum displacement as a function of initial along-track distances in the absence of errors in the predicted positions
- An assumption which allows along-track errors in the region of possible conflict to be considered as being equivalent to errors in the initial along-track distances
- Calculation of the distribution of the minimum displacement
- Calculation of the probability of conflict

3.1 Minimum displacement as a function of initial along-track distances

Consider an encounter between two aircraft X and Y in the horizontal plane.

Suppose that the tracks are straight in the region where a conflict may occur (although they may contain turns before and after this region).

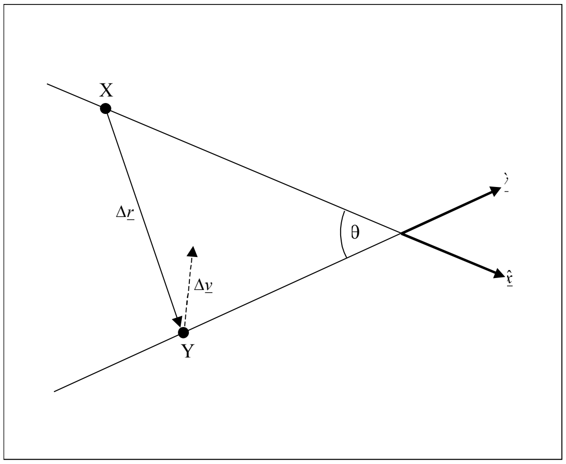

Figure 1: Encounter geometry

Let

O	be the crossing point
$\hat{\underline{x}}$, $\hat{\underline{y}}$	be unit vectors along the tracks of X and Y
x' , y'	be the initial along-track distances of X and Y with respect to the crossing point
$\underline{r}_X$, $\underline{r}_Y$	be the positions of X and Y With respect to O $\underline{r}_X = x'\hat{\underline{x}}$, $\underline{r}_Y = y'\hat{\underline{y}}$
$\underline{v}_X$, $\underline{v}_Y$	be the velocities of aircraft X and Y $\underline{v}_X = v_X \hat{\underline{x}}$, $\underline{v}_Y = v_Y \hat{\underline{y}}$
θ	be the crossing angle

The initial position of aircraft Y relative to aircraft X is given by

$$\Delta \underline{r} \equiv \underline{r}_Y - \underline{r}_X = y'\hat{\underline{y}} - x'\hat{\underline{x}} \qquad (1)$$

The velocity of aircraft Y relative to aircraft X is given by

$$\Delta \underline{v} \equiv \underline{v}_Y - \underline{v}_X = v_Y \hat{\underline{y}} - v_X \hat{\underline{x}} \qquad (2)$$

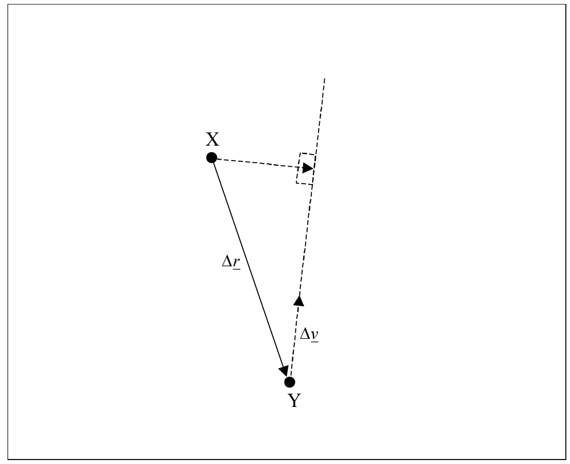

Figure 2: Relative position and velocity view

Relative to X, Y begins at its initial relative position and moves with time in the direction of the relative velocity. When the distance between them is a minimum, the relative position vector is perpendicular to the relative velocity. This is the case regardless of the initial positions of the aircraft. (It is also worth noting that the minimum distance depends only upon the direction of the relative velocity and not upon its magnitude.)

Let $\underline{k}$ be a unit vector perpendicular to the plane containing the tracks. For convenience, let $\underline{k}$ be in the same direction as $\hat{\underline{x}} \times \hat{\underline{y}}$. Let $\Delta\hat{\underline{v}}$ $\left(\equiv \Delta\underline{v} / |\Delta\underline{v}|\right)$ be the unit vector in the direction of the relative velocity. The vector $\Delta\hat{\underline{v}} \times \underline{k}$ is therefore a unit vector in the plane, perpendicular to the relative velocity. When the distance between X and Y is a minimum, the <u>signed</u> distance in the direction $\Delta\hat{\underline{v}} \times \underline{k}$ (hereafter termed the minimum displacement) can be found by resolving the initial relative position vector in that direction, i.e.,

$$d_{\min} = \Delta\underline{r} \bullet \left(\Delta\hat{\underline{v}} \times \underline{k}\right) \qquad (3)$$

This is a scalar triple product and its value is unaffected by a cyclic reordering of the three vectors, so that

$$d_{\min} = \underline{k} \bullet \left(\Delta\underline{r} \times \Delta\hat{\underline{v}}\right)$$

$$= \underline{k} \bullet \left(\left(y'\hat{\underline{y}} - x'\hat{\underline{x}}\right) \times \left(\frac{v_Y \hat{\underline{y}} - v_X \hat{\underline{x}}}{|\Delta\underline{v}|} \right) \right)$$

$$= \frac{(y'v_X - x'v_Y)(\underline{k} \bullet (\hat{\underline{x}} \times \hat{\underline{y}}))}{|\Delta \underline{v}|} \tag{4}$$

where $|\Delta \underline{v}|$ is the magnitude of the relative velocity.

The unit vector $\underline{k}$ was defined to be in the direction $\hat{\underline{x}} \times \hat{\underline{y}}$, so that $\underline{k} \bullet (\hat{\underline{x}} \times \hat{\underline{y}}) = \sin\theta$. Consequently, the minimum displacement is given by

$$d_{\min} = \frac{(y'v_X - x'v_Y)\sin\theta}{|\Delta \underline{v}|} \tag{5}$$

Note that the minimum displacement is a simple linear combination of the initial along-track distances.

The magnitude of the relative velocity is given by

$$|\Delta \underline{v}|^2 = \Delta \underline{v} \bullet \Delta \underline{v}$$

$$= (v_Y \hat{\underline{y}} - v_X \hat{\underline{x}}) \bullet (v_Y \hat{\underline{y}} - v_X \hat{\underline{x}})$$

or

$$|\Delta \underline{v}| = \sqrt{v_Y^2 - 2 v_X v_Y \cos\theta + v_X^2} \tag{6}$$

Consequently, the minimum displacement

$$d_{\min} = \frac{(y'v_X - x'v_Y)\sin\theta}{\sqrt{v_Y^2 - 2 v_X v_Y \cos\theta + v_X^2}} \tag{7}$$

Dividing numerator and denominator by v_X

$$d_{\min} = \lambda(y' - mx') \tag{8}$$

where $m \equiv v_Y / v_X$ is the ratio of the speeds of the aircraft

and $\lambda \equiv \frac{\sin\theta}{\sqrt{m^2 - 2m\cos\theta + 1}}$

Alternatively, the minimum displacement can be expressed as a function of the difference in the times of arrival of the aircraft at the crossing point:

$$d_{\min} = \lambda v_Y \Delta\tau$$

where $\Delta\tau = \frac{y'}{v_Y} - \frac{x'}{v_X}$ is the difference in the times of arrival at the crossing point.

Or $|d_{\min}| = \lambda v_Y |\Delta\tau|$

For given speeds and angle of approach, the minimum distance and the difference in times of arrival at the crossing point are directly proportional. As a consequence, the maximum rate at which aircraft can be passed through a fixed crossing point is inversely proportional to the minimum distance between them.

3.2 An assumption concerning along-track errors

Aircraft flight management systems (FMS) can control cross-track navigational errors to within very small tolerances compared with along-track errors, which are not controlled by the FMS. For this reason, and for simplicity, only along-track errors are considered in this paper. (The effect of cross-track errors on the minimum displacement can be handled in a similar way to along-track errors - see [Irvine, 2002]. It is useful to be able to handle both kinds of error as other sources of error in predicted position, for example due to the timing of instructions given by a controller or the timing of the initiation of manoeuvres by a pilot, may be broken down into along-track and cross-track components). The effect of an along-track error is to advance or retard an aircraft along its track. The along-track error is acquired as the aircraft progresses along its track, so that in reality its speed will differ from the predicted speed and is not constant. It is assumed, however, that within the range of along-track distances for which conflict is possible (the region of possible conflict), the along-track error in an aircraft's position is approximately constant and that the aircraft flies with its predicted speed. This assumption is also made in [Paielli and Erzberger, 1997].

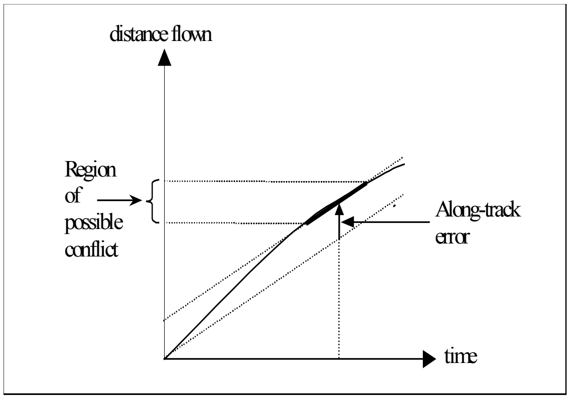

Figure 3: An assumption

Having made this assumption, we can consider an alternative process by which an aircraft might acquire a given along-track error, namely, that the error is present in the initial along-track distance of the aircraft and that it then flies with constant speed. Subject to this assumption, an observer watching the movement of an aircraft within the region of possible conflict cannot distinguish which process gave rise to the along-track error.

Let the error in the predicted position of aircraft X along its track at a time τ into the future be a random variable denoted by $\alpha_X(\tau)$. Similarly let the error in the predicted position of aircraft Y along its track be a random variable denoted by $\alpha_Y(\tau)$ Subject to the above assumption, these errors can be considered as being equivalent to errors in the initial along-track distances:

$$
\begin{aligned}
x' &= x_0 + \alpha_X(\tau) \\
y' &= y_0 + \alpha_Y(\tau)
\end{aligned}
\quad (9)
$$

Since the along-track errors $\alpha_X(\tau)$ and $\alpha_Y(\tau)$ are random variables so too are x' and y'. Similarly, the minimum displacement, as given by equation (8), is also a random variable. Substituting the above expressions for the 'equivalent' initial along-track distances (taking along-track errors into account) gives:

$$d_{\min} = \lambda\left[\left(y_0 + \alpha_Y(\tau)\right) - m\left(x_0 + \alpha_X(\tau)\right)\right]$$

which can be separated into a nominal and a random term:

$$d_{\min} = \lambda\left[\left(y_0 - mx_0\right) - \left(\alpha_Y(\tau) + m\alpha_X(\tau)\right)\right] \quad (10)$$

3.3 Distribution of along-track error and minimum displacement

According to [Paielli and Erzberger, 1997, Paielli, 1998], the along-track distance error at a time for aircraft in straight and level flight is well modelled by a normal distribution. This statement effectively summarises the cumulative effect over time of an underlying stochastic process, without saying anything about its time-varying characteristics. We can write

$$\alpha(\tau) = a(\tau) A_\tau \quad (11)$$

where A_τ is a normally distributed random variable with zero mean and unit variance and $a(\tau)$ models the growth of the standard deviation of the along-track error with time.

If the along-track errors are normally distributed, then the minimum displacement (10) becomes a sum of normally distributed variables, which is itself normally distributed. This is the case even if the component variables are correlated.

The mean μ of a sum of independent normally distributed variables is the sum of the individual means, and the variance σ^2 is the sum of the individual variances. The mean is the nominal minimum displacement in the absence of errors in predicted positions. (A linear combination of correlated normal variables can be expressed as a linear combination of uncorrelated normal variables if the covariance matrix is known.)

For the purposes of illustration, assuming that the along-track errors are independent and have the same variance, and neglecting cross-track errors, the mean and variance of the minimum displacement are given by

$$\mu = \lambda\left(y_0 - mx_0\right)$$

$$\sigma^2 = \lambda^2 a(\tau)^2\left(1 + m^2\right)$$

and the standard deviation is given by

$$\sigma = \lambda a(\tau)\sqrt{1+m^2}$$

or $$\sigma = \gamma a(\tau) \quad \text{where } \gamma \equiv \lambda\sqrt{1+m^2}$$

(12)

It is interesting to investigate how the variance of the minimum displacement depends upon the angle of approach and the ratio of the speeds of the aircraft.

$$\sigma^2 = \gamma^2 a(\tau)^2 \quad \text{where}$$

$$\gamma^2 = \lambda^2\left(1+m^2\right)$$

$$= \frac{\sin^2\theta\left(1+m^2\right)}{1-2m\cos\theta+m^2}$$

$$= \frac{\left(\frac{1}{m}+m\right)\sin^2\theta}{\left(\frac{1}{m}-2\cos\theta+m\right)}$$

The value of γ^2 is unaffected by replacing m with $1/m$. A graph of γ^2 against angle of approach is shown below for various values of the ratio of the speeds of the aircraft.

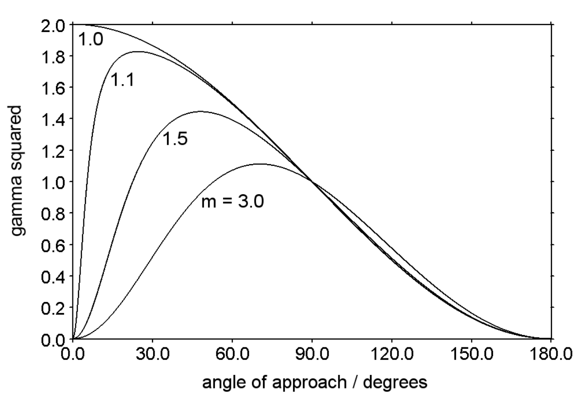

Figure 4: Minimum displacement variance against angle of approach for various ratios of speed

γ^2 is zero for an angle of approach of 180 degrees. For aircraft approaching in opposite directions along parallel tracks along-track errors have no effect on the minimum distance. As the angle of approach θ decreases from 180 degrees to 90 degrees γ^2 increases monotonically. In this range of angles of approach the value of γ^2 depends little on the ratio of the speeds. When $\theta = 90$ degrees, $\gamma^2 = 1$ for all ratios of the speeds. In this case, the variance of the minimum displacement is due entirely to the variance of the along-track error of one aircraft. As the angle of approach decreases from 90 degrees towards zero γ^2 rises to a maximum value, which is always greater than or equal to 1. The greater the difference in the speeds, the greater the angle at which the maximum is reached. In the general case that the speeds of the aircraft are different γ^2 tends to zero for small angles of approach. In the special case that the speeds are identical, the maximum value of γ^2 occurs when the angle of approach is zero, and is equal to 2 – the variances of the along-track errors of both aircraft simply sum to give the variance of the minimum displacement.

According to [Paielli and Erzberger, 1997, Paielli, 1998], for prediction intervals of up to 20 minutes, the standard deviation of the along-track distance error grows linearly with time, i.e., $a(\tau) = \dot{a}\tau$ where $\dot{a}$ is the rate of growth of the standard deviation of the along-track error. In this case

$$\sigma = \gamma\dot{a}\tau \qquad (13)$$

For aircraft in level flight along-track error grows at a rate of about 0.25 nautical miles per minute.

3.4 Probability of conflict

The probability of conflict corresponds to the area underneath the minimum displacement distribution that lies between minus the required separation and plus the required separation. This is illustrated below for a probable conflict.

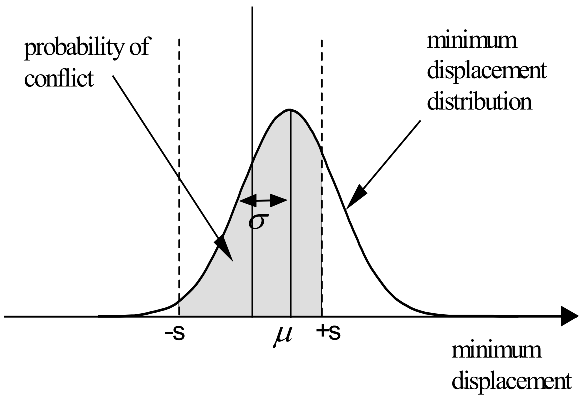

Figure 5: A probable conflict

$$P_{conflict} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-s}^{+s} \exp\left(-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2\right) du$$

$$= \frac{1}{\sqrt{2\pi}} \int_{(-s-\mu)/\sigma}^{(+s-\mu)/\sigma} \exp\left(-z^2/2\right) dz \qquad (14)$$

or

$$P_{conflict} = \Phi\left(\frac{+s-\mu}{\sigma}\right) - \Phi\left(\frac{-s-\mu}{\sigma}\right)$$

(15)

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-z^2/2\right) dz$

is the cumulative probability[1] or distribution function of a normally distributed random variable with a mean of zero and variance of one (and can be evaluated using a pre-computed table and interpolation). $\Phi(x)$ is shown below:

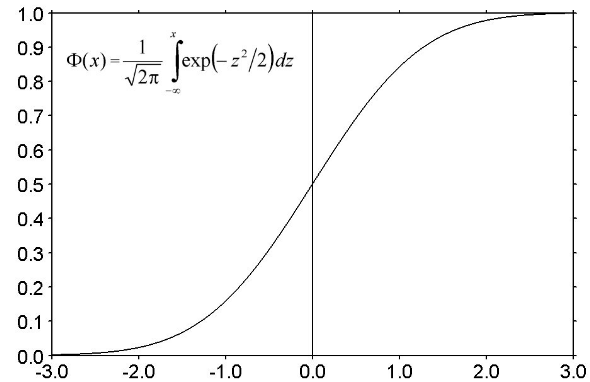

Figure 6: Cumulative probability function

The reader is referred to [Irvine 2002] for further details discussion of this approach to conflict probability estimation.

4. An approximation for probability of conflict for resolved conflicts

Figure 5 showed the minimum displacement distribution for a probable conflict. Most of the area beneath the distribution lies in the range $(-s,+s)$.

Now suppose that the probable conflict is solved horizontally by making speed or heading changes to one or both of the aircraft. The effect of this to move the mean of the distribution, that is, the nominal minimum displacement, such that the bulk of the distribution lies outside of the range $(-s,+s)$. (The change will also have some impact on the width of the distribution.) The minimum displacement distribution after resolution will now look something like the following:

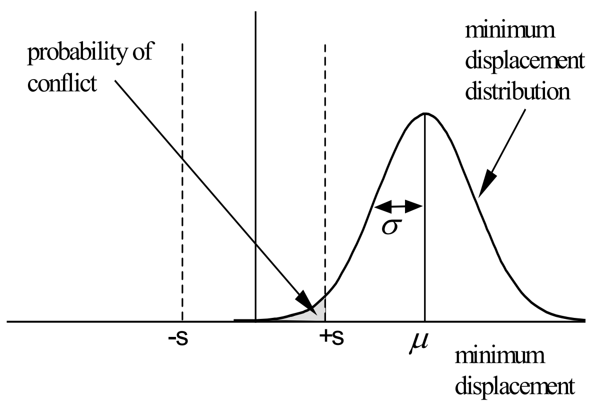

Figure 7: After resolution

[1] The cumulative probability function $\Phi(x)$ of a random variable X is the probability that X takes a value less than or equal to x, i.e., $\Phi(x) = P(X \le x)$. For a continuous random variable the cumulative probability function $\Phi(x)$ is the area beneath the probability density function $\phi(x)$ over the interval $(-\infty, x)$.

Consider the case that the mean is positive and lies to the right of the range $(-s,+s)$, as shown above. Three regions can be identified on this graph: if the actual value of the minimum displacement is greater than $+s$ then the potential conflict is solved in the way it is intended that it be solved (the actual value of the minimum displacement has the same sign as the nominal minimum displacement); if the actual value of the minimum displacement lies inside the range $(-s,+s)$ then conflict occurs; if the actual value of the minimum displacement is less than $-s$ then the potential conflict is solved but not in the way it is intended that it be solved (the actual value of the minimum displacement has the opposite sign from the nominal minimum displacement. For example, the intention was to pass an aircraft in front of another, but in fact it passed behind but with more than the required separation). The area beneath the minimum displacement distribution in each of these three regions is the probability of each of these cases arising.

Consider the expression for probability of conflict in equation (15)

$$P_{conflict} = \Phi\left(\frac{+s-\mu}{\sigma}\right) - \Phi\left(\frac{-s-\mu}{\sigma}\right)$$

The second term has a physical interpretation – it is the probability of the potential conflict being solved, but counter to the way in which it was intended that it be solved. We expect this probability to be small for well-solved conflicts. Provided that the mean lies more than one standard deviation to the right of the range $(-s,+s)$, i.e. $\mu > s + \sigma$, and that the standard deviation is less than the width of the range, i.e. $\sigma < 2s$, then effectively the whole of the "tail" of the distribution lies in the range $(-s,+s)$. In this case the second term in equation (15) is small compared with the first and can be neglected, giving the following approximation for the residual probability of conflict following resolution by changing the relative velocity:

$$P_{\substack{conflict,\\ after\_resolution}} \approx \Phi\left(\frac{+s-\mu}{\sigma}\right) \qquad (16)$$

for $\mu > s + \sigma$, $\sigma < 2s$

In the symmetrical case that the mean is negative and lies more than one standard deviation to the left of the range $(-s,+s)$, i.e. $\mu < -s - \sigma$, it can be shown that

$$P_{\substack{conflict,\\ after\_resolution}} \approx \Phi\left(\frac{+s+\mu}{\sigma}\right) \qquad (17)$$

for $\mu < -s - \sigma$, $\sigma < 2s$

These two expressions can be combined to give an expression for positive and negative values of the mean:

$$P_{\substack{conflict,\\ after\_resolution}} \approx \Phi\left(\frac{+s-|\mu|}{\sigma}\right) \qquad (18)$$

for $|\mu| > s + \sigma$, $\sigma < 2s$

Note that $|\mu|$ is the nominal miss distance.

This approximation to the probability of conflict (thick line) is illustrated below for 90 degree crossing conflicts with nominal miss distances from 6 to 12 nautical miles (using the parameters specified in section 4.3), together with the probability of conflict (thin line) calculated using both terms in equation (15).

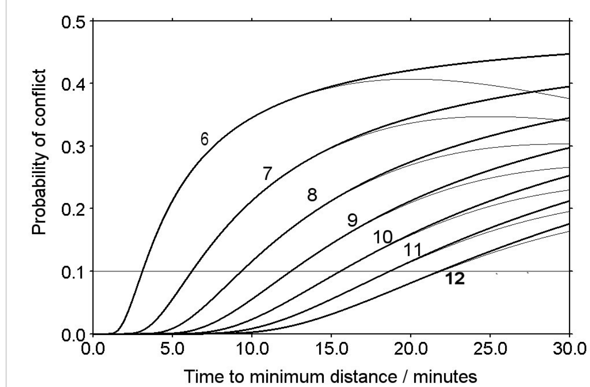

Figure 8: Approximation to probability of conflict

It can be seen that for well-solved conflicts (residual probability of conflict < 0.1), the approximation is excellent. Note also that where there is a discrepancy the approximation overestimates the probability of conflict. As discussed above, the discrepancy is the probability that the potential conflict will be solved but counter to the way in which it was intended that it be solved.

5. Achieving a required conflict probability

A required probability of conflict can be achieved by choosing appropriate values of various parameters.

5.1 By choosing a sufficiently large nominal miss distance

What are we trying to achieve when solving conflicts? The further the mean of the minimum displacement distribution is moved away from the range $(-s,+s)$, the lower the probability of conflict between the two aircraft concerned. How far should the mean be moved? One answer is that it should be moved sufficiently that the probability of conflict is lowered to a required level. Let the required probability of conflict be ε. In other words, after resolution there remains a probability ε that further intervention will be needed to solve the conflict.

Using the approximation derived above we can write

$$\varepsilon \approx \Phi\left(\frac{+s-|\mu|}{\sigma}\right) \qquad (19)$$

Rearranging this,

$$|\mu| \approx s+k\sigma \qquad (20)$$

where $k=-\Phi^{-1}(\varepsilon)$

In words, to achieve a residual probability of conflict of ε the nominal miss distance must exceed the required separation by k standard deviations where $k=-\Phi^{-1}(\varepsilon)$. The correspondence between the probability of conflict and the number of standard deviations by which the nominal minimum distance must exceed the required separation can be read from the graph of $\Phi(x)$ in figure 6. Some selected values are tabulated below.

Required probability of conflict after resolution ε	Number of standard deviations by which the nominal minimum distance must exceed the required separation $k=-\Phi^{-1}(\varepsilon)$
0.16	**1.00**
0.10	1.28
0.02	**2.00**
0.01	2.33

Table 1

From equation (20) it is clear that, for a required probability of conflict, the margin by which the nominal miss distance must exceed the required separation is proportional to the standard deviation of the minimum displacement distribution. The standard deviation of the minimum displacement is proportional to the standard deviation of the along-track prediction error, which grows with time (equation 12). Consequently, for a given required probability of conflict, the margin by which the nominal miss distance exceeds the required separation must grow with time in the same way as the standard deviation of the along-track prediction error.

$$|\mu| \approx s+k\sigma = s+k\gamma a(\tau) \qquad (21)$$

If the standard deviation of the along-track error grows linearly with time (see earlier) then:

$$|\mu| \approx s+k\sigma = s+k\gamma \dot{a}\tau \qquad (22)$$

where

s is the required separation

k is the number of standard deviations by which the nominal minimum distance must exceed the required separation, and corresponds to the required probability of conflict

γ depends upon the angle of approach and the ratio of the speeds of the aircraft, and is less than or equal to $\sqrt{2}$

$\dot{a}$ is the rate of growth of the standard deviation of the along-track error (of one aircraft)

τ is the time interval into the future at which minimum distance is reached

This in effect gives a nominal miss distance rule that takes account of the above parameters.

(If the growth of the nominal miss distance is linear in time then this has an interesting consequence for the implementation of conflict detectors. Trajectories are often modelled as sequences of straight-line segments. In a geometric conflict detector the detection of potential conflicts involves considering pairs of segments, one from each trajectory. Time intervals during which separation will be lost are found by solving an equation that is quadratic in time and in which the required separation appears as a constant. If this constant is replaced by a term that is linear in time, the equation remains quadratic. In other words, a simple geometric conflict detector that uses a constant required separation could be modified to detect encounters in which the probability of conflict exceeds a given value.)

5.2 By initiating resolution manoeuvres sufficiently late

If we wish to work with a given value of the nominal miss distance, we can calculate the time before minimum distance at which the resolution manoeuvre should begin in order to achieve a required probability of conflict:

$$\tau \approx \frac{|\mu| - s}{k\gamma \dot{a}} \qquad (23)$$

5.3 By using a sufficiently accurate trajectory predictor

If we know the time before minimum distance at which we wish to initiate manoeuvres and can also specify the nominal miss distance, we can calculate the rate of growth of along-track error which would be needed in order to achieve a given probability of conflict. This could be taken as a required trajectory predictor performance.

$$\dot{a} \approx \frac{|\mu| - s}{k\gamma \tau} \qquad (24)$$

6. Illustrations

In the following illustrations, except where stated otherwise, the rate of growth of the standard deviation of the along-track position error is that given in [Paielli and Erzberger 1997] for aircraft in level flight, i.e. 0.25 nautical miles per minute. One would expect the rate of growth of along-track error to be greater for aircraft that are climbing or descending. Both aircraft have the same speed, and the required separation is taken to be 5 nautical miles.

6.1 Nominal miss distances to achieve a required probability of conflict for various manoeuvre initiation times

The following graph shows distributions of the minimum displacement for 90 degree crossing encounters with different manoeuvre initiation times such that a probability of conflict of 0.1 is achieved. In 10% of such cases further intervention would be needed to prevent the required separation from being breached. It can be seen that the further ahead one looks in time the wider and flatter the distribution becomes, and the greater the nominal miss distance (the mean of the distribution) must be to achieve the required probability of conflict. To achieve a probability of conflict of 0.1 the nominal miss distance must exceed the required separation by 1.28 standard deviations (see table 1).

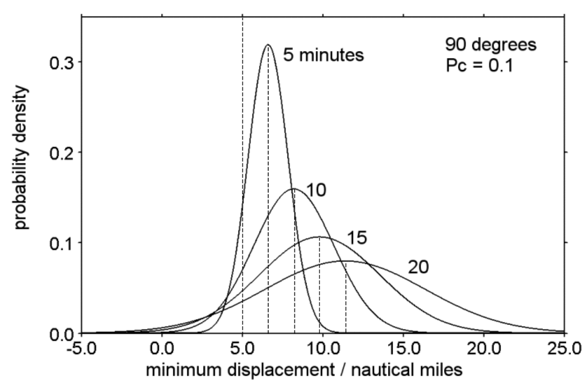

Figure 9: Minimum displacement distributions

The next graph shows how the nominal minimum distance needed to achieve a probability of conflict of 0.1 varies with angle of approach, again with various manoeuvre initiation times between 5 and 20 minutes.

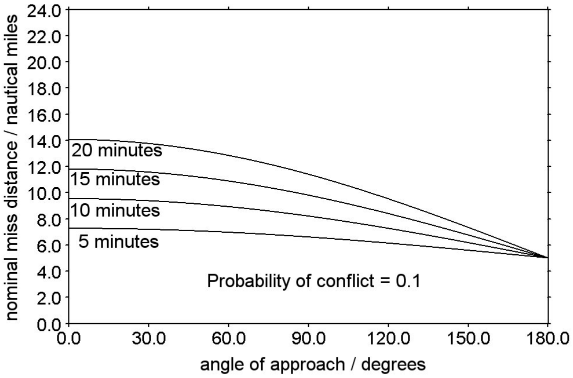

Figure 10: Nominal miss distance, $P_{conflict} = 0.1$

In the case that the speeds of the aircraft are identical, that is, $m = 1$, $\lambda = \cos(\theta/2)$, and the cosine dependency on (half) the angle of approach is apparent in this graph.

For the following graph the parameters are identical with the exception that the required probability of conflict after resolution is chosen to be 0.01, so that in only 1% of cases would further intervention be needed to prevent the required separation from being breached. In this case the nominal miss distance must exceed the required separation by 2.33 standard deviations (see table 1).

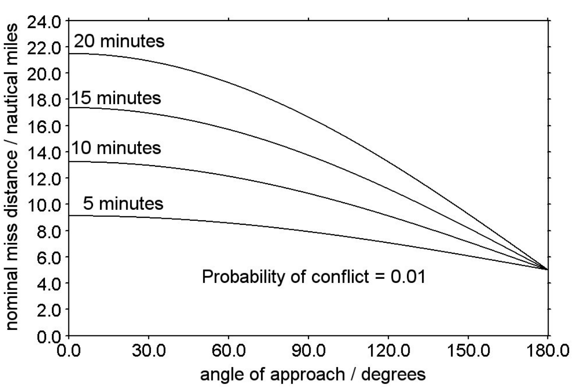

Figure 11: Nominal miss distance, $P_{conflict} = 0.01$

It is apparent that, for aircraft in level flight with a rate of growth of along-track error of 0.25 nautical miles per minute, for long resolution time horizons and low required probabilities of conflict the target miss distance can be large compared with the required separation. For example, for manoeuvre initiation times of 20 minutes, nominal miss distances of up to 14 nautical miles would be needed to achieve a probability of conflict of 0.1, and nominal miss distances of up to 22 nautical miles would be needed to achieve a probability of conflict of 0.01. One would expect the rate of growth of along-track error to be greater for aircraft that are climbing or descending. As discussed earlier, the maximum rate at which aircraft can be passed through a given crossing point is inversely proportional to the nominal miss distance, so that increased nominal miss distances would decrease the rate at which aircraft could be passed through a crossing point while achieving a required probability of conflict.

6.2 Manoeuvre initiation times to achieve a required probability of conflict for various nominal miss distances

The following graph shows the earliest times at which manoeuvres should be initiated in order to achieve a probability of conflict of 0.1 for various nominal miss distances.

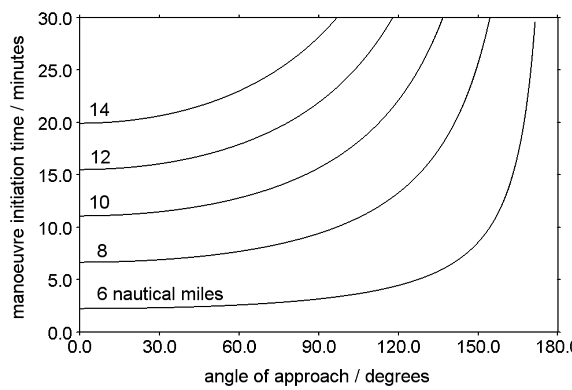

Figure 12: Earliest manoeuvre initiation time

6.3 Rate of growth of along-track error needed to achieve a required probability of conflict

The following graph shows the rate of growth of along-track prediction error which must be attained in order to achieve a probability of conflict of 0.1 with a nominal miss distance of 10 nautical miles, for a range of manoeuvre initiation times. For a given nominal miss distance, the greater the manoeuvre initiation time the lower the rate of growth of along-track error must be to achieve a required probability of conflict.

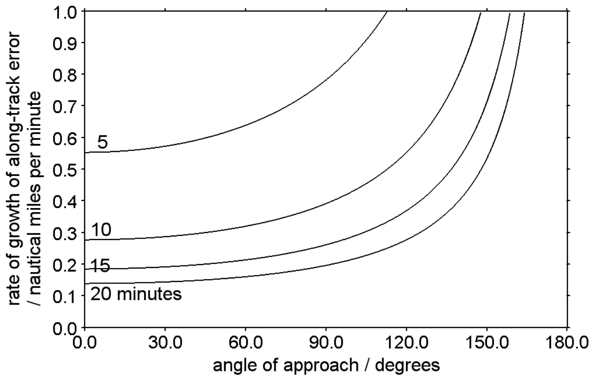

Figure 13: Required rate of growth of along-track error

7. Conclusions

In short-term tactical control (2 to 5 minutes) it is reasonable to talk of potential conflicts as having been solved or not solved. In the medium-term, where along-track prediction errors are greater, it makes more sense to talk about the quality of a resolution, in particular, in terms of the residual probability of conflict. As a consequence, operational concepts should cater for the need to update some horizontal resolutions.

A simple relationship was derived between nominal miss distance, required probability of conflict and the magnitude of the error in the predicted positions of the aircraft involved.

Possible trade-offs between nominal miss distance, manoeuvre initiation time and trajectory prediction accuracy were illustrated.

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