

# A LINEAR PROGRAMMING APPROACH FOR ROUTE AND LEVEL FLIGHT ASSIGNMENT

*Dritan Nace, Jacques Carlier, Nhat Linh Doan, Heudiasyc Laboratory, UMR CNRS 6599, University of Technology of Compiègne, Centre de Recherches de Royallieu, BP 20 529, F-60205 Compiègne cedex, France.*

*Vu Duong, Eurocontrol Experimental Centre, Centre de Bois des Bordes, BP15, F-91222 Brétigny sur Orge cedex, France.*

## Abstract

This study concerns the route and level flight assignment problem aiming at global flight plan optimization, which has already become a key issue owing to the growth of air traffic. Better coordination of all existing flights for all airlines is becoming an increasingly desirable goal. A number of related problems appear in the operations research literature, notably vehicle routing, scheduling and other transportation problems. Several studies have been especially devoted to the problem of aircraft scheduling and routing. Aircraft routing requires the generation of non-colliding, time-dependent routes through a specified airspace that we call the airspace network. The problem considered here can be modeled as a specific flow problem in a given space-time network. This study aims at estimating the effects of routing capabilities at a quantitative level (the congestion level, i.e. the number of potential en-route conflicts), and at a qualitative level (traffic smoothing). We present a deterministic model based on a *Linear Programming* approach for optimizing the level route assignment in a trajectory-based Air Traffic Management (ATM) environment. This problem can be seen as a multi-period (dynamic) problem where the time dimension is an essential ingredient to consider when constructing flight plans. This dynamic problem can be transformed into a static one by using standard technique of time-expanding the underlying network. We propose here a model to consider the airspace congestion in a finer way: we consider the number of aircraft involved in potential en-route conflicts rather than the number of aircraft in a sector, sometimes implicitly understood as en-route capacities in ATM.

## 1. Introduction

The flight route and level assignment problem, aiming at global flight plan optimization, has already become a key issue owing to the growth of air traffic. There is expected to be an increase of 5% in air traffic each year, so that traffic in 2015 will be double that of

1998. Better coordination of all existing flights for all airlines is particularly necessary, given this growth. We address the problem of global flight plan optimization from a routing point of view (assigning the appropriate route and level to each flight) in a given airspace network. Our aim is to reduce the number of potential en-route conflicts. This problem is obviously very close to those addressed by ATFM. Indeed, the role of ATFM at EEC is to ensure that air traffic does not overtake the capacity of airports and ATC (Air Traffic Control) sectors. In Europe, this function is achieved by the CFMU (Central Flow Management Unit). When demand is higher than capacity in some ATC sectors, traffic must be regulated by delaying some flights through the slot allocation process. Another mean is to re-route some flights in agreement with the airlines concerned. As we explain later, we do study the problem of optimization of route and level assignment, but in a sectorless environment (not the environment defined by ATC sectors).

We present here a mathematical model for this problem. We propose the use of dynamic network flow models, which, for our requirements, can accurately represent this problem. Such methods are already known in the operations research area, but we have introduced some changes in accordance with the objectives and constraints of the problem under consideration. In this paper we briefly review the dynamic network flow model of Ford and Fulkerson ([1]) and Ahuja et al ([2]), and discuss its application to ATM. At this stage, we have tried to take advantage of our experience in routing in telecommunication networks [3], in order to develop a suitable model for ATM. We propose a model which is particularly suitable in the *sectorless* (see [4] for further details on ATM *sectorless* concept) airspace context: indeed, we intend to assign an appropriate route to each flight in order to avoid potential en-route conflicts and make easy in this way the end-to-end control of the flight.

## ***Contribution of This Work***

In this paper we propose an approach that intends to optimize the route and flight-level assignment in a trajectory-based ATM environment. This model is an extendable one. Indeed, it seems easy to integrate slot-allocation requirements in the future. Note that this work is an extension of this presented in [5]. In contrast, we are concentrated here on resolving simultaneously the level and route assignment and providing computational results for realistic data size. Finally we propose a heuristic to handle large instances in real time.

This paper is organized as follows. After this introduction, in section 2 we give a brief review of related works on this problem. Then in section 3, we describe the mathematical model. Some experimental results are given in section 4. In section 5, we discuss the interest and limits of the model and describe a heuristic that permits to consider larger instances. Section 6 is devoted to some concluding remarks.

## **2. Background**

### ***Related Works in the ATM and Operations Research Area***

A number of related problems appear in the operations research literature, notably vehicle routing, scheduling and other transportation problems. Several studies are devoted to the problem of aircraft scheduling and routing. Aircraft routing has been subject of studies presented in [6], [7], [8], [9], [10], [11], [12], [13], [14], etc. Literature concerned with ground holding one (often presented as a scheduling problem) is also abundant but this problem is beyond the scope of the paper. Let us first cite Bertsimas and Stock ([6], [7]) who have considered the *Air Traffic Flow Management Rerouting Problem* (TFMRP), treating simultaneously the time and route assignment problems through an efficient deterministic approach. Delahaye and Odoni (see [8]) have considered this problem from a stochastic optimization point of view. They propose a genetic algorithm to resolve the whole problem.

Interesting works have been done also on the level assignment problem. So Letrouit in [11], Barnier and Brisset in [10] have considered the problem of flight level allocation to aircraft flows in order to ensure their separation. This can be stated as a graph-coloring problem, known to be NP-hard. Note that these studies are confined to predefined direct routes, so there is a single fixed route for each flight. Furthermore, each flight has been seen as a flow (occupying the entire route

and not as an “object” in motion). Hence, the “time” factor has not really been taken into account. In our study we intend to define a dynamic linear programming model to determine both routes and flight levels. Notice finally that the level assignment problem is quite similar to the wavelength routing in telecommunication optical networks.

Throughout these studies modeling variations can be distinguished, such as deterministic versus stochastic, and static versus dynamic models.

As for airspace network design, an interesting methodology has been reported in [15]. This study relies on Voronoy diagrams for modeling the network. The method described allows nodes to be located in the network with respect to ATC requirements and cost considerations. We must also cite [16], which gives two relevant network-based models for Air Traffic Control.

Other interesting studies using linear programming techniques are presented in [1], [17], and more particularly the Dantzig-Wolfe and Benders decomposition for aircraft routing and crew scheduling, [18], and for multi-period routing in [19]. Even though these works are not directly related to our problem, they provide applications of the sort of fundamental decomposition approach that we intend to use for solving larger instances.

Studies in the operations research area, more precisely flows in networks (see [2], [20]), are particularly interesting and useful for a deeper study of our problem. Indeed, aircraft routing requires the generation of non-colliding, time-dependent routes through a specified airspace that we call the airspace network. So the problem considered in this project can be modeled as a specific flow problem in a given space-time network. The primary difficulty is how to model such a network. Indeed, network and aircraft flow are in motion, and we are used to representing the network in the static case. So the first thing to do is to reduce this “dynamic flow network” to a static one. This problem was first addressed in [1]. Dynamic network flow has since been used by many researchers in modeling various vehicle routing and scheduling problems. In [14], the authors show how the dynamic network flow modeling can be used to compute how to route as many aircraft as possible through the network (max-flow problem). But our problem is quite different: we need to route a given set of flights under a given set of fixed economic and safety requirements. Consequently, the associated LP model is also different. It is clear, however, that all models used for route computation are of great interest with regard to the problem in hand. We

can see, for instance, that our problem concerns route computing under time and cost constraints known as constrained shortest path problem, which is already shown to be NP-hard.

Our objectives are quite similar to these in ATFM optimisation studies. The ATFM programme includes the elaboration and validation of Standard Routing Schemes, as well as the study of slot allocation mechanisms. The goal is to improve current approaches and to develop new algorithms. Let us cite projects as CARAT (Computer Aided Route Allocation Tool, intended to develop flow management re-routing prototypes to assist in the management of the CFMU route or COSAAC (COMmon Simulator to Assess ATFM Concepts), a simulation tool used in the pre-tactical phase and AMOC (ATFM Modeling Capability) which is another EEC ATFM simulator. Finally, SRS (Standard Route Scheme) is a strategically planned routing system designed to make the most effective use of ATC capacity. It enables ATC to maximise capacity by defining routes that provide an organised system of major traffic flows through congested areas, and reducing the crossing of major flows at critical points. Our study is not limited to developing a re-routing tool. We hope that it could be useful in the elaboration of strategic and tactical ATFM planning tools. In the following we present a deterministic model intended to optimize simultaneously route and flight-level assignment in a trajectory-based ATM environment. This model is intended to represent the airspace congestion in a more accurate way: we consider the number of aircraft involved in potential en-route conflicts. As far as we know, airspace congestion has only ever been considered in terms of *en-route capacities* (the number of aircraft in each sector), and has not explicitly been studied from the point of view of the number of en-route conflicts on segments of air routes. We remark that our model is particularly suitable in the trajectory-based ATM airspace context: indeed, we intend to assign an appropriate route to each flight in order to avoid potential en-route conflicts and thus facilitate the end-to-end control of the flight. Lastly, one should note that this model is extensible, with the possibility of future integration of slot-allocation requirements.

### 3. Modeling and Mathematical Formulation

The model presented here is a linear programming one, more precisely a 0/1 Linear Programming because of existence of some binary variables. We have chosen

the arc-path formulation as the most appropriate one to model this kind of problem. The advantages of this formulation (see [20]), are obviously related to the use of variables corresponding directly to flight-routes. Indeed, we think that the alternative flow models (using mass balance or Kirchoff constraints) are not suitable in case of a flow problem with constrained paths, reducing so the number of eligible routes, as it is the case in Air Traffic networks.

#### *Designing the Airspace Network*

Several assumptions are necessary to set the topology of the airspace network. The main idea is to *remodel the existing network*. The network starts off with a number of initial nodes, corresponding to beacons, and a set of fixed links corresponding to the probable used links. This can be realized simply by deducing the “used” links from a given air traffic situation, where all flight-plans are given, and preferred routes are already known. At this stage, we add all nodes associated with the crossing points of existing links which are potential en-route conflicts. In the graph corresponding to the initial network, potential conflicts can be seen as the overload of some nodes, identified as potential conflict areas. An efficient way to measure this load is to replace all potential conflict nodes by links whose load is easier to measure. Note that we do not consider congestion in airports in this study, even if they are represented in the network. Links located physically very near each other, which according to specialists correspond to the same potential conflict area, could be also replaced by a unique link. The duration of one time period is fixed according to the desired level accuracy of disposed data.

#### *Modeling*

Let us first fix some realistic hypotheses which will help us to build our initial model:

1. *aircraft are in constant motion;*
2. *a number of aircraft may be placed in the same air segment, and conflicts resolved using appropriate method, i.e. there is no capacity imposed on links;*
3. *takeoff and landing times are given and not changeable and they all reach their destination, i.e. aircraft respect some given slot allocation;*
4. *flight level is assigned once for all, changes en-route are not considered at this stage.*

Notice that the last two hypothesis are placed temporarily, we shall reconsider them when we deal with a generalized model in the near future. In the following we detail our deterministic model. It is based on a linear programming approach for optimizing the route assignment in a trajectory-based ATM environment, essentially intended to reduce airspace congestion. The aim of this model is route-computation with load balancing: more precisely the reduction of potential en-route conflicts. It can easily be seen that the route assignment problem is a multi-period (dynamic) one which can be transformed into static one by using the standard technique of time-expanding the underlying network. In [1], Ford and Fulkerson showed how to transform the maximum dynamic flow problem into a conventional network flow problem. Let us consider in the following how in our problem the dynamic flow problem can be reduced to a conventional network flow problem. The advantage lies in being able to represent a “dynamic” network by a static one.

In a dynamic network flow problem, arcs have traversal times and flow received at an intermediate node must be moved immediately. Notice that generally capacities are associated with arcs but for our problem we do not consider capacities (see the hypothesis 2)). Now, we need to represent the flow that have to be moved from the source,  $s$ , to the sink,  $t$ , within some given period of time  $T$ , and to find how to route it according to some predefined objectives. The reduction uses the given network,  $G(V,E)$ , to build a new network  $G^*(V^*,E^*)$ , as follows: for each node,  $v \in V$ , periods in  $[1,T]$ , create  $T$  copies:  $v_1, \dots, v_T$ , and in addition, set up a super-source,  $s^*$ , and a super-sink,  $t^*$ . Let  $t(v, w)$  be the traversal time of arc  $(v, w)$ ; introduce an arc from  $v_i$  to  $w_j$  (of capacity equal to that of arc  $(v, w)$  when such exists) if and only if  $(j - i) = t(v, w)$ , (where  $(j - i)$  gives the time between these two periods). Finally, connect the super-source to every copy of the original source, and connect every copy of the original sink to the super-sink, all with arcs of unbounded capacity. Note that these reductions create a new graph that is  $T$  times larger than the original one. It can easily be seen that the route assignment problem is a multi-period dynamic one. Indeed, the time dimension is an essential ingredient to consider when constructing flight plans for a large number of flights, and evaluating the number of potential en-route conflicts. Notice that all co-existing flights, especially those involved in the same cluster of en-route conflicts, generally have different origins, destinations, duration, and takeoff and landing times. Obviously, this dynamic problem can be transformed into a static one by using the standard technique of time-expanding the

underlying network as described above. We therefore reduce the time horizon (typically consisting of 24 hours or longer) to a finite number of discrete periods, and replicate the network for each period. Each replication can be seen as a snapshot of the network at the corresponding period.

In our model we typically divide the time horizon into discrete periods of a few minutes. However, the duration of a period may be adjusted depending on factors such as the accuracy of the available data and modeling granularity. We also assume that there is at most one flight for a given pair (O, D) during each single time period. It will be noticed that this problem is closely related to the time-slot allocation problem, but in the latter case, the increased complexity leads us to consider the take-off time of aircraft fixed once and for all. We also take the network as given. An interesting study is presented in [4], suggesting a fundamentally different control method: end-to-end control instead of local control (ATC sectors). With this method, controllers are responsible for all flights on certain routes. As we can see, our model is particularly suited to the *sectorless environment* context: we assign an appropriate route and level to each flight in order to avoid potential en-route conflicts and thus facilitate the end-to-end control of the flight.

### ***Notation and Mathematical Formulation***

- Let  $G(V, A)$  be a directed (static) network defined by a set  $V$  of nodes ( $v$ ) and a set  $A$  of directed arcs ( $k$ ).
- Let  $A'$  be a specified set of arcs corresponding to potential en-route conflict areas.
- Let  $T$  be the set of periods  $t$  and  $L$  the set of allowed flight levels  $l$ .
- Let  $F$  be the set of flights to be routed. For each flight  $f$ , we suppose the origin/destination and the period of taking-off to be known.  $F(t)$  gives the set of flights  $f$  taking-off at period  $t$ .
- $H(f)$  denotes the set of eligible routes (preferred routes, supposed known) for a given flight  $f$ . For each route, its trajectory is supposed known.
- $L(f)$  denotes the set of eligible levels (preferred levels) for a given flight  $f$ .
- $x_{j,fl}^t$  gives the traffic value (0/1) using the route  $j$  for the flight  $f$ , level  $l$ , taking-off at period  $t$ .
- $c_{j,fl}^t$  denotes the cost associated with the route  $j$  for the flight  $f$ , level  $l$ , taking-off at period  $t$ .
- $y_{k,l}^t$  corresponds to the number (decreased by one) of aircraft flying through arc  $k$  at flight level  $l$  during the period  $t$ .

- $a_{j,f,k,l}^p$  generally takes value in  $[0, 1]$  according to the probability for flight  $f$ , route  $j$ , flight-level  $l$ , at period  $p$  after the taking-off, to be at link  $k$ . At this stage we have limited the choice for values of  $a_{j,f,k,l}^p$  in only 0 or 1.
- $R$  gives the maximum number of aircraft involved in the same conflict.

We have assumed that the flight-level is fixed once for all and no level changes are considered.

This problem can be mathematically stated as below:

**Minimize**

$$R + \alpha \sum_{k \in A', t \in T, l \in L} y_{k,l}^t + \beta \sum_{\substack{t \in T, f \in F(t), \\ j \in H(f), l \in L(f)}} (c_{j,f,l}^t x_{j,f,l}^t)$$

**Subject to:**

(1)  $\forall t \in T, \forall k \in A', \forall l \in L, y_{k,l}^t + 1 \leq R;$

(2)  $\forall t \in T, \forall f \in F(t), \sum_{j \in H(f), l \in L(f)} x_{j,f,l}^t = 1;$

(3)  $\forall t \in T, \forall k \in A', \forall l \in L,$

$$\sum_{f \in F(t'), t' < t, j \in H(f)} (a_{j,f,k,l}^{t-t'} x_{j,f,l}^{t'}) \leq y_{k,l}^t + 1;$$

(4)  $\forall t \in T, \forall f \in F(t), \forall j \in h(f), \forall l \in L(f), x_{j,f,l}^t \text{ binary};$

(5)  $\forall t \in T, \forall k \in A', \forall l \in L, y_{k,l}^t, R \in \mathbb{R};$

where  $x_{j,f,l}^t$  give the decision variables.

The objective function tends to minimize firstly the maximum number of aircraft flying simultaneously at the same airspace, (see also constraints (1)). The second term of the objective function tends to minimize globally the number of potential conflicts and the third one evaluates the global cost induced by chosen routes.  $\alpha$  and  $\beta$  give the adjusting coefficients in accordance with the importance of the corresponding term.

The set of constraints (2) and (4) enforces a unique route per flight. The set of constraints (3) gives the occupation of some airspace through the route at time  $t$ . We "count" only the effective conflicts, (involving more than one aircraft). We suppose that for a given route and a given taking-off time, we can predict the location of the aircraft in the airspace after any elapsed time. In other words, we suppose known all eventual trajectories which can be determined by: the route (2D), the level flight, the velocity (known) and the departure time (given). This is done through the values given to  $a_{j,f,k,l}^p$

during the preliminary phase of preferred routes selection.

The problem addressed in this paper is formulated as an integer linear programming problem with binary variables, and to find an exact solution is not an easy task. Data are not necessarily accurate, and we must deal with the uncertainty in the delays. For instance, for a long haul flight we cannot know exactly the location of the aircraft after several hours of flight. At first glance it would also appear that the model cannot consider the general case for aircraft with different flying speeds. In reality, our model can to a certain extent handle these two problems. Indeed, it is sufficient to fix for each trajectory the approximate time and the duration (or periods) where the aircraft will be in each potential conflict area, which takes some account of the speed of the aircraft and the uncertainty. Obviously, this can be done by fixing appropriately the  $a_{j,f,k,l}^p$  coefficients, *i.e.* if in the normal situation the aircraft doing flight  $f$ , route  $j$ , flight-level  $l$ , taking off in period  $t$ , is predicted to be on link  $k$  during the period  $t+p$ , we fix  $a_{j,f,k,l}^p = 1$ , and in order to take into account the uncertainty we can also fix  $a_{j,f,k,l}^{p-1} = 1$  and/or  $a_{j,f,k,l}^{p+1} = 1$ .

## 4. Numerical Results

The model presented above has already been implemented. All testing data, provided by EUROCONTROL, correspond to two instances: a real French air transportation network European air transportation Network on 12 August 1999. We have varied the granularity time for 5 and 10 minutes and we have run tests with each network. Tests instances are called NET\_FR\_5, NET\_FR\_10 for the French network and NET\_EU\_5, NET\_EU\_10 for the European network with respect to the size of period.

In Table 1 we represent some characteristics of the considered test instances. We have considered a restricted number of preferred routes for each flight, (typically 3). We have assumed that the number of preferred levels is also limited to 3: the preferred level and the appropriate next upper and below levels. The departure time is also given.

**Table 1: Description of Test Cases.**

Test Instances	Number of Flights	Used Airports	Used WayPoints
NET_FR	1697	134	769
NET_EU	22427	831	5010

Actually, the computing program is composed of two parts:

- Network modeling and preferred route generation through a k-shortest path algorithm.
- Computing the optimized route and level assignment (CPLEX).

In Table 2 we give the size of the problem resolved by MIP of CPLEX. The algorithm was implemented in C++ and using the ILOG MIP solver CPLEX 8.0. All of tests were run on a machine with the following configuration: Pentium IV 2.0 GHz, 512MB Ram, Windows XP.

We report in Table 3 the computation time of each test case. The column “MIP solver” shows the elapsed time for solving the problem to optimality with CPLEX MIP solver. The next column gives the time for modeling the problem and solving it. However, we do not take into account the time for constructing the network and generating the routes, which means only the calculation time for the second part of the program is considered. In summary, when using the period of 5 or 10 minutes, we are able to find an optimal integral solution for European network problem, which consists of 291641 constraints and 201246 binary variables (22427 flights), in less than 2 hours.

Table 4 shows the performance of the approach in improving the congestion situation. We have compared the maximal number of aircraft in conflict and the global number of conflicts for both cases, with standard and optimized routes and we have remarked significant gains as shown in the table. More precisely, the “max cluster size” in first two columns gives the maximum number of flights in a conflict on the same arc. The “avg. conflict” was calculated by counting the number of flights

involved in conflicts (an aircraft could possible be involved in several non-simultaneous conflicts) and dividing by the total number of flights. In Figures 1 and 2 we have compared the number of potential conflict points for periods of 5 and 10 minutes. Note that we have supposed that the aircraft velocity is constant for both situations, which explains the large number of conflicts in given real air traffic situations. However these results show the high potential in improving the congestion situation through an appropriate route and level assignment.

**Table 2: Description of Corresponding MIP.**

Network	Constraints	Variables	Binary Vars
NET_FR_5	11509	20126	15219
NET_FR_10	14741	21742	15219
NET_EU_5	218117	201246	201246
NET_EU_10	291641	335854	201246

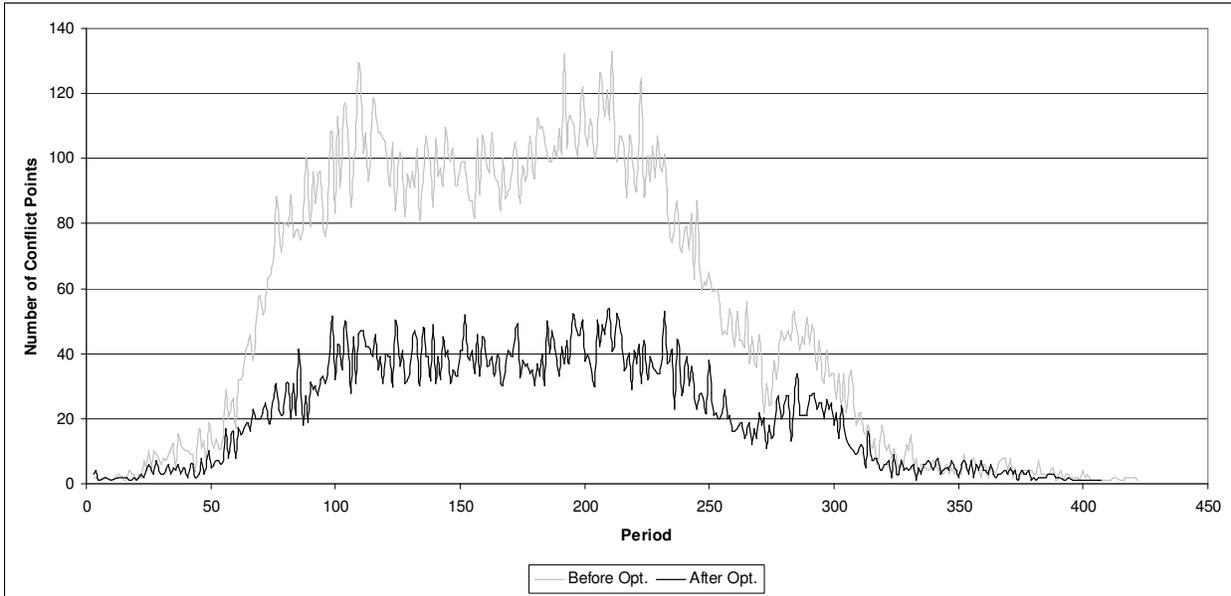
**Table 3: Computation Time (sec).**

Network	MIP Solver	Model & Solve
NET_FR_5	59.00	67.00
NET_FR_10	151.00	162.00
NET_EU_5	219.00	3893.00
NET_EU_10	330.00	6025.00

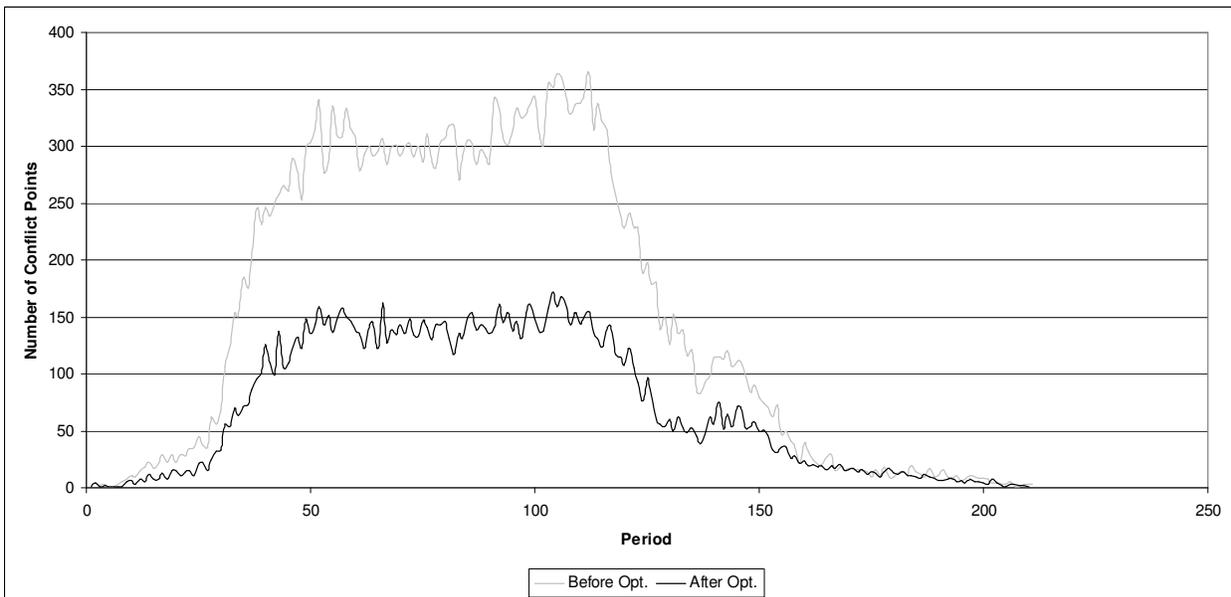
**Table 4: State of Conflicts Before and After Optimization.**

Network	Max Size of Cluster With Initial Flights	Max Size of Cluster With Opt. Flights	Avg. Conflicts (Before Opt.)	Avg. Conflicts (After Opt.)	Gain	Conflict Points Before Opt.	Conflict Points After Opt.	Gain
NET_FR_5	4	2	1.19	0.52	56.26%	963	469	51.30%
NET_FR_10	5	2	1.32	0.68	48.21%	1028	584	43.19%
NET_EU_5	7	2	2.01	0.77	61.50%	21074	8681	58.81%
NET_EU_10	8	3	3.20	1.42	55.64%	31985	15054	52.93%

**Figure 1: Comparison of Number of Conflict Points Before and After Optimization (period of 5 minutes)**



**Figure 2: Comparison of Number of Conflict Points Before and After Optimization (period of 10 minutes)**



## 5. Discussion

The time granularity is of great importance for our model. As explained above, the size of the dynamic network models is proportional to the number of periods, and it becomes greater as the size of the period is small. In contrast with the latter remark the problem becomes more combinatory

while increasing the size of periods. This can be simply explained by the fact that the conflicts “become” more frequent. We have remarked that Binary Linear Programming method described in section 3 approaches its limits when the size of periods is greater than 15 minutes. Heuristics are also envisioned i.e. relaxing and rounding, aggregating, etc. After a number of tests we have observed that the relaxed solution is not always a good starting point to

build a good quality solution for our problem. In contrast, the level of optimization depends essentially on the “quality” of preferred routes. Furthermore, our model is “path-oriented”, but as the number of paths is greater, the combinatorics increase and it renders the problem more difficult. So it seems necessary to look for better route generation method. We propose in the following to keep separated the route generation computation from route and level assignment. The first task can be realized by resolving a multi-commodity multi-period flow problem and the second one through an iterative algorithm. In the following we describe briefly this heuristic.

### Sketch of heuristic:

I). Resolving a multi-commodity multi-period flow problem. *A flow represents aggregation of flights with the same source, destination.* Obtaining a set of a limited number of candidate routes

#### II). Route and level assignment

- Resolving the problem for each flight-level. All possible flights for each level are considered.
- Associate some penalty with each flight.
- Remove the most penalizing flights for each level.
- Repeat until there is a single level assigned to each flight.

Two main ideas reside behind this heuristic. First, we think that the number of “quality” routes is quite restricted mostly for economical reasons, which justify the idea of proceeding in two separated stages. Second, considering the network level by level (see step II of the heuristic), seems logical as flights are in conflict only with these sharing the same flight-level. Some numerical results reporting the performances of the heuristic will be given in the conference.

### Equitability

The model presented in section 3 could be slightly modified in order to handle the “equitability”. We recall that this model is intended to *Sectorless ATM environment* where the traffic control is done on routes instead of sectors. So, we need to balance the workload of controllers through an equitable repartition of conflicts through routes and overall reducing of the number of simultaneous conflicts on the same route controlled by a controller. For this, we have modified the model given section 3. First, we introduce new binary variables  $z_{k,l}^t$ , which state the

existence of a conflict on arc  $k$ , period  $t$  and level  $l$ . Some new set of constraints are also added and finally we modify appropriately the objective function as follows:

### Minimize $Z$

Subject to:

$$(6) \quad \forall t \in T, \forall f \in F(t),$$

$$\sum_{j \in H(f), l \in L(f)} x_{j,f,l}^t = 1 ;$$

$$(7) \quad \forall t \in T, \forall k \in A', \forall l \in L, \quad N z_{k,l}^t \geq y_{k,l}^t ;$$

$$(8) \quad \forall t \in T, \forall k \in A', \forall l \in L,$$

$$\sum_{j \in H(f), f \in F(t'), t' < t} (a_{j,f,k,l}^{t-t'} x_{j,f,l}^{t'}) \leq y_{k,l}^t + 1 ;$$

$$(9) \quad \forall t \in T, \forall f \in F(t), \forall j \in H(f),$$

$$\sum_{k \in j, l \in L(f)} z_{k,l}^t \leq Z ;$$

$$(10) \quad \forall t \in T, \forall k \in A', \forall l \in L, \quad z_{k,l}^t \text{ binary};$$

$$(11) \quad \forall t \in T, \forall f \in F(t), \forall j \in h(f), \forall l \in L(f), \quad x_{j,f,l}^t, \text{ binary};$$

$$(12) \quad \forall t \in T, \forall k \in A', \forall l \in L, \quad y_{k,l}^t, R \in \mathbf{R}^+;$$

Where  $N$  gives an upper bound of the maximum number of aircraft involved in the same conflict, (assumed known).

## 6. Concluding Remarks and Future Work

This study is intended to reduce the airspace congestion through an appropriate route and level flight assignment. This problem has been modeled and resolved through Linear Programming methods and some encouraging results are already reported. We recall that we handle in someway the uncertainties through fixing appropriately the time granularity.

We can remark that this problem is related closely to the slot-time allocation one: extending the model with departure scheduling time variation is possible. This model should also take into account the connecting flights. So, our aim is proposing a "generic" model that we will complete in the near future in order to consider the global flight plan optimization problem: route, level and time assignment for each flight. Another interesting

problem is how to adapt this approach for operational needs.

## References

[1] Ford, L. R., and D. R. Fulkerson., 1958 *Constructing maximal dynamic flows from static flows*. Operations Research, 6:419-433.

[2] Ahuja R.K., T.L. Magnanti and B. Orlin, 1993, *Network Flows: Theory, Algorithms and Applications*, Prentice Hall.

[3] Nace D., 2002, *A linear programming based approach for computing optimal splittable fair routing*, ISCC'02, Taormina-Giardini Naxos, Italy.

[4] Duong V., G. Gawinowski, J.P. Nicolaon and D. Smith, 2001, *Sector-Less Air Traffic Management*, The 4th ATM R&D Seminar, Santa Fe, USA.

[5] Nace D., J. Carlier, N-L. Doan and V. Duong, *Using Dynamic Flow Network Modeling for Aircraft Route Assignment*, the 21<sup>st</sup> Digital Avionics Systems Conference (DASC), Irvine, California, USA, 27-31 October 2002.

[6] Bertsimas D.J. and A.R. Odoni, 1998, *The air traffic flow management problem with en-route capacities*, Operations Research, Vol. 46, pp. 406-422

[7] Bertsimas D. and A. Odoni, 2000, *The traffic flow management rerouting problem in Air Traffic Control: A dynamic Network Flow Approach*, Transportation Science © 2000 INFORMS, Vol. 34 No. 3, pp. 239–255.

[8] Delahaye D. and A. Odoni, 1997, *Airspace Congestion Smoothing by Stochastic Optimization*, Evolutionary Programming VI, pp. 163--176.

[9] Serali H.D., J.C. Smith and A. Trani, 2000, *An Airspace Planning Model for Selecting Flight-Plans under Workload, Safety and Equity Considerations*, The Third International Air Traffic Management R&D Seminar ATM-2000, Napoli, Italy.

[10] Barnier N. and P. Brisset, 2002, *Graph Coloring for Air Traffic Flow Management*, Proceedings CPAIOR'02, pp. 1-15.

[11] Letrouit V. *Optimisation du Réseau des Routes Aériennes en Europe*, PhD Disertation, INPG, 1998.

[12] Sherry J. E., C.G. Ball, S.M. Zobell, *Traffic Flow Management (TFM) weather Rerouting*, The 4th ATM R&D Seminar, Santa Fe, USA.

[13] Nilim A, L El Ghaoui, V. Duong, M. Hansen, *Trajectory-based Air Traffic Management (TB-ATM)*

*under Weather Uncertainty*, The 4th ATM R&D Seminar, Santa Fe, USA.

[14] Chen Z, A. T. Holle, B. Moret, and J. Saia, A. Boroujerdi, *Network Routing Models Applied to Aircraft Routing Problems*. Proc. Winter Simulation Conference, Arlington, USA, ACM, pages 1200-1206, 1995.

[15] Mehadhebi K., 2000, *A Methodology for the Design of a Route Network*, The Third International Air Traffic Management R&D Seminar ATM-2000, Napoli, Italy.

[16] Zenios S.A., 1991, *Network Based models for air traffic control*, European Journal of Operational Research, Vol. 50, pp. 166-178.

[17] Tosic V., 1997, *Some Models & Algorithms for En-Route Air Traffic Flow Management*, The Third International Air Traffic Management R&D Seminar, Saclay, France.

[18] Cordeau J.F., G. Stojkovic, F. Soumis and J. Desrosiers, 2000, *Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling*, Les Cahiers du GERAD.

[19] Martin V., J.L. Lutton and J. Geffard, 1998, *Multiperiod Capacity Planning model for global telecommunication networks*, NETWORKS'98, Sorrento, Italie, pp. 261-266.

[20] Minoux M., 1983, *Programmation mathématique, théorie et algorithmes*, Vol. 2, Eds. Dunod.

## Keywords

Air Traffic Flow Management, trajectory-based environment, linear programming, routing.

## Biography

**Jacques Carlier** received the degrees of Computer Science in 1972 and Mathematics in 1974, MSc (DEA) in Computer Science in 1973, PhD (Doctorat de 3ème cycle) in Disjunctive Scheduling Problems in 1975 and PhD (Doctorat d'Etat) in Resource Constrained Scheduling in 1984, all from University of Paris VI. From 1974 to 1985 he was Assistant Professor at the same University. Since 1985 he is Professor at University of Technology of Compiègne. He is currently director of the MSc in Computer Science at the same university. His research interests include operations research and in particular, scheduling problems, reliability and routing in telecommunication networks.

**Vu Duong** is currently Head of the Innovative Research at the EUROCONTROL Experimental Center. Prior to this position, he had been FREER Project Manager, ESCAPE Project Manager, then Simulator Development Program Manager at EEC. Vu holds a Master degree in Engineering and a Ph.D. degree in Artificial Intelligence, both from Ecole Nationale des Ponts et Chaussées, Paris, France.

**Nhat Linh DOAN** is graduated in Computer Science in 1996 from University of Technology of HoChiMinh VietNam, and receive the MSc (DEA) degree in Computer Science in 2001 from IFI (Institut de la Francophonie pour l'Informatique). He is currently preparing his PhD thesis at University of Technology of Compiègne on the problem of network global max-min fairness and air traffic flow management. His research interests include operations research and in particular, network routing, optimization.

**Dritan Nace** received the degrees of Mathematics in 1991 (University of Tirana, Albania), MSc (DEA) in Computer Science in 1993 and PhD in Computer Science in 1997, both from University of Technology of Compiègne. From 1997 to 1998 he worked as research engineer at CNET (Centre National d'Etudes des Télécommunications), the France Telecom research center. Since 1998, he is Assistant Professor at the University of Technology of Compiègne. His research interests include operations research and more particularly network design and optimization, routing etc.