Air Traffic Complexity based on Non Linear Dynamical Systems

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Abstract

This paper present a new air traffic complexity metric based on non-linear dynamical systems. The goal of this metric is to identify any trajectory organisation in the traffic pattern in order to quantify the associated control difficulty. Many others works have proposed metrics in the past, but they usually identify one feature of the complexity and were not able to address any pattern organization. A full vector field can be summarized by the equation of a dynamical system which describe and control the evolution of a given state vector ($\vec{X} = [x, y, z]^T$). The key idea of our work is to find a dynamical system which modelizes a vector field as close as posible to the observations given by the aircraft positions (and speeds). Two approaches are then presented. The first one is based on a linear dynamical system and produce an aggregate complexity metric. The second one, which is the main part of this paper, uses a non-linear dynamical system modeling which fits the observations without error. Such a modeling enable to identify high (low) complexity areas on a map and addresses trajectory segments instead of vector field which is more relevant for the air traffic management application (the air controller sees speed vectors on his screen but works on trajectory segments in his mind in order to produce resulution scenario (past and future)). A collocation techniques has been used to speed up the computation of the associated complexity metric in order to address large areas with many aircraft. Such a metric is very adapted to compare different traffic situations for any scale (sector or country).

Keywords : Complexity, Dynamic Systems, Topological Entropy, Control Workload.

1 Introduction

Air traffic control organizes air flows in order to ensure flight safety and to increase the capacity of the route network. Currently, about 7500 are registered everyday over France, which is a crossroad for the whole European airpsace. This traffic generates a huge amount of control workload and the airspace is then divided into elementary sectors which are managed by air navigation controllers. For several years, a constant increase of air traffic has induced more and more congestion in the control sectors. Two strategies can then be applied to reduce such a congestion. The first one consist in adapting the demand to the existing capacity (slot-route allocation, collabarative decision making etc...). The second one adapts the capacity to the demand (modification of the air network, of the sectorization, new airportsetc ...). For the two preceding approaches, the capacity of a sector is measured by the number of aircraft flying across the sector during a given period of time. The observation of controlled sectors shows that sometimes, controllers accept planes beyond the capacity threshold, while in other situations, they refuse traffic before the capacity is reached. This phenomenon clearly shows that the operational metric alone cannot account for controller's workload. The goal of this study is to synthesize a traffic complexity indicator in order to better quantify the congestion in air sector, which will be more relevant than a simple number of aircraft which is independent of the traffic configuration. More precisely, our objective is to build some metrics of the intrinsic complexity of the traffic distribution in the airspace. Those metrics must capture the level of disorder (or organization) of any traffic distribution. Usuallly, metrics are focused on the speed vector distribution and the associated disorder metric captures only some features of the traffic complexity. The real objective of our work is to build a metric which measures the disorder or the organization of a set of trajectories in a 4D space (3D for the space and 1D for the time).

Those metric are relevant for many applications in the air traffic management area. For instance, when a sectoring is designed [3], the sectors have to be balanced from the congestion point of view and for the time being, only the number of aircraft is used to reach this objective. Another example where a congestion metric is needed is the traffic assignment [2, 4] for which an optimal time of departure and a route are searched for each aircraft in order to reduce the congestion in the airspace. Complexity metric may also be used to design new air networks, for dynamic sectoring concept, to define future ATM concepts (Free Flight) etc....Complexity metrics would enable to qualify and quantify the performance of the Air Traffic service providers and enable a more objective consultation between airlines and providers.

The work presented in this paper is based on the dynamical systems modeling of the air traffic. A dynamical system describes and controls the evolution of a given state vector. If such a vector is given by the position of aircraft $\vec{X} = [x, y, z]^T$, a dynamical system associates a speed vector $\dot{\vec{X}} = [vx, vy, vy]^T$ to each point in the airspace. So, a full vector field can be summarized by the equation of the dynamical system. The key idea is to find a dynamical system which modelize a vector field as close as possible to the observations given by the aircraft positions (and speeds). Based on this dynamical system modeling, a trajectory disorder metric can be easily computed.

In a first part, this paper will summarize the previous related works. The second part will present a linear dynamical system modeling for which the complexity metric can be represented into a complex coordinate system. In this system, it is very easy to identify any speed vector organization pattern. The third part introduces a non linear extension of the previous dynamical system modeling. Such a modeling is exact and can fit the observations without error. Such a non linear modeling can be used to produce maps of complexity by identifying areas with high(low) complexity. This extension can also address time extension of the model and can then work straightly on the trajectory segments instead of speed vectors. A collocation techiques has been used to speed up the computation of the associated complexity; mainly when such a computation is done on large areas like countries.

2 Previous related works

The airspace complexity is related with both the structure of the traffic and the geometry of the airspace. Different efforts are underway to measure the whole complexity of the airspace.

Significant research interest in the concept ot ATC complexity was generated by the "Free Flight" operational concept. Integral to Free Flight was the notion of dynamic density. Conceptually, dynamic density is a measure of ATC complexity that would be used to define situations that were so complex that centralized control was required [15].

Windemere inc [11] proposed a measure of the perceived complexity of an air traffic situation.This measure is related with the cognitive workload of the controller with or without knowledge of the intents of the aircraft. The metric is human oriented and is then very subjective.

Laudeman et al from NASA [13] have developed a metric called "Dynamic Density" which is more quantitative than the previous one and is based on the flow characteristics of the airspace. The "Dynamic Density" is a weighted sum of the traffic density (number of aircraft), the number of heading changes (¿15), the number of speed changes (¿0.02 Mach), the number of altitude changes (¿750 ft), the number of aircraft with 3-D Euclidian distance between 0-25 nautical miles, the number of conflicts predicted in 25-40 nautical miles. The parameters of the sums have been adjusted by showing different situations of traffic to several controllers. Finally, B.Sridhar from NASA [16], has developed a model to prodict the evolution of the metric in the near future. Efforts to define "Dynamic Density" have identified the importance of a wide range of potential complexity factors, including structural considerations.

The traffic itself is not enough to describe the complexity associated with an airspace. A few previous studies have attempted include structural consideration in complexity metrics, but have done so only to a restricted degree. For example, the Wyndemere Corporation proposed a metric that included a term based on the relationship between aircraft headings and dominant geometric axis in a sector [11]. The importance of including structural consideration has been explicitly identified in recent work at Eurocontrol. In a study to identify complexity factors using judgment analysis, "Airspace Design" was identified as the second most important factor behind traffic volume [12].The impact of the structure on the controller workload can be found on the paper [9, 10]. Those papers show how strong the structure of the traffic (airways, sectors, etc...) is related with the control workload.

The previous models do not take into account the intrinsic traffic disorder which is related to the complexity. The first efforts related with disorder can be found in [6]. This paper introduces two classes of metrics which measure the disorder of a traffic pattern. The first class is based on geometrical properties and proposed new metrics which are able to extract features on the traffic complexity such as proximity (measures the level of aggregation of aircraft in the airspace), convergence (for close aircraft, this metric measures how strongly aircraft are closer to each other) and sensitivity (this metric measure how the relative distance between aircraft is sensible to the control manouver). The second class is based on a dynamic system modeling of the air traffic and use the topological entropy as a measure of disorder of the traffic pattern.

G.Aigoin has extended and refined the geometrical class by using a cluster based analysis [1]. Two aircraft are said to be in the same cluster if the product of their relative speed and their proximity (a function of the inverse of the relative distance) is above a threshold. For each cluster, a metric of relative dependence between aircraft is computed and the whole complexity of the cluster is then given by a weighted sum of the matrix norm. Those norms give an aggregated measure of the level of proximity of aircraft in clusters and the associated convergence. From the cluster matrix, it is also possible to compute the difficulty of a cluster (it measures how hard it is to solve a cluster). Multiple clusters can exist within a sector, and their interactions must also be taken into account. A measure of this interaction has been proposed by G.Aigoin [1]. This technique allows multiple metrics of complexity to be developed such as average complexity, maximum and minimum cluster complexities, and complexity speeds.

Another approach based on fractal dimension has been proposed by S.Mondoloni and D, Liang in [14]. Fractal dimension is a metric comparing traffic configurations resulting from various operational concepts. It allows in particular to separate the complexity due to sectorization from the complexity due to traffic flow features. The dimension of geometrical figures is well-known: a line is of dimension 1, a rectangle of dimension 2,etc.... Fractal dimension is simply the extension of this concept to more complecated figures, whose dimension may not be an integer. The block count approach is a practical way of computing fractal dimensions: it consists in describing a given geometrical entity in a volume divided into blocks of linear dimension d and counting the number of blocks contained in the entity N.The fractal dimension D_0 of the entity is thus :

$$D_0 = \lim_{d \to 0} \frac{\log N}{\log d}$$

The applicatin of this concept to air route analysis consists in computing the fractal dimension of the geometrical figure composed of existing air routes. An analogy of air traffic with gas dynamics then shows a relation between fractal dimension and conflict rate (number of conflicts per hour for a given aircraft). Fractal dimension also provides information on the number of degrees of freedom used in the aispace: a higher fractal dimension indicates more degrees of freedom. This information is independent of sectorization and does not scale with traffic volume. Therefore, fractal dimension is a measure of the geometrical complexity of a traffic pattern.

Some new geometrical metrics have been developed in [5]which are able to capture the level of disorder or the level of aorgainization for some traffic patterns. For instance, in a curl moving, the speed vector are very different even if the global moving is full organized without any changes in the relatives distance between aircraft. To capture those features, the covariance and the Koenig metrics have been developed. The first one is able to identify disorder or organization of translation movings. The second one identify organized curl moving.

All the previous metrics capture only one feature of the complexity and are not able to produce an aggregate metric which can capture all the possible situations (high-low density, how-low convergence, translation organization, curl organization etc ...). The topological entropy (kolmogorov entropy) is the only metric which is able to capture most features of the complexity. The non-linear form is even able to identify any trajectories organizations (aircraft following the same path at the same speed). The next section will describe this metric in detail for the linear and the non-linear forms.

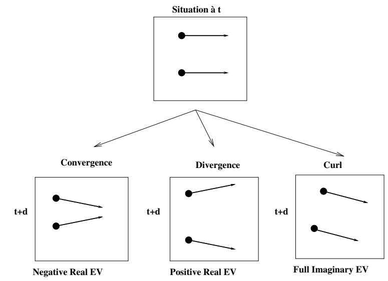

Figure 1: Impact of the eigen values on the dynamic of a system

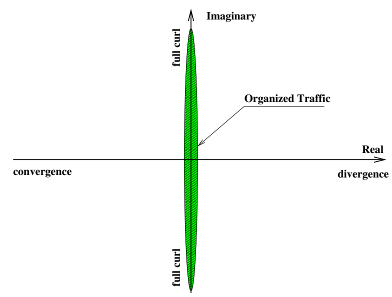

Figure 2: Impact of the eigen values on the dynamic of a system

3 Linear Dynamical System Modeling

3.1 Principle

This metric is based on the modeling of the set of trajectories by a linear dynamic system. This enable to identify different structure of organization of the aircraft speed vectors such as translation, curl organizations or a mix of them.

The key idea of this metric is to model the set of aircraft trajectories by a linear dynamical system which is controlled by the following equation :

$$\dot{\vec{X}} = A.\vec{X} + \vec{B}$$

where $\vec{X}$ is the state vector of the system :

$$\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The eigen values of the matrix A control the evolution of the system. The real part of those eigen values is related with the convergence or the divergence property of the system in the direction of the eigen vector. When such a eigen value has positive real part, the system is in expansion mode and when it is negative the system is in contraction mode. On the other end, the imaginary part of the eigen values are related with the level of curl organization of the system. Depending of those eigen values, a dynamical system can evolve in contraction, expension, rotation or a combination of those three modes. The figure 1 shows examples of evolutions with different eigen values. The initial situation consist in two aircraft moving in parallel at the same speed and three possible evolutions are then drawn after a time period d. In the first case, the eigen value is real and negative; the system evolves in a contraction mode and the two aircraft are converging. The second situation represent a diverging evolution for which the eigen value is real and positive. It must be noticed that in the two previous situations, the distance between aircraft changes with time. The last evolution is associated with a full imaginary eigen value for which the aircraft stay a the same distance from each other in a curl moving.

The evolution properties of the system related with the position of the eigen values can be summerized in the complex coordinate system (see figure 2).

In this coordinate system, the vertical ellipse identifies the organized situations (in translation, curl or both).

A dynamical system can then be considered as a map T from the state space $\mathcal{X}$ to itself. Based on this mapping T, one can define the topological entropy (or Kolmogorov entropy) of the dynamical system which measure the level of mixing of $\mathcal{X}$ by T. This entropy is associated with the changes of the relative distances between points from $\mathcal{X}$ by T. The topological entropy of a dynamical system is then a disorder indicator of the distribution of the aircraft in the considered airspace. This entropy is not a statistical metric and is very adapted to the air traffic control system for which there are few aircraft in the sectors.

3.2 Kolmogorov Entropy Computation

The kolmogorov entropy is computed with the help of the eigen value decomposition of the matrix A :

$$A = L.S.U^T$$

where S is the diagonal matrix of the eigen values. Based on the observations of the aircraft (positions and speed vectors), the dynamical system has to be adjusted with the minimum error. This fitting has been done with a Least Square Minimization method. For each considered aircraft i, it is supposed that position $\vec{X_i} = [x_i, y_i, z_i]^T$ and speed vector $\vec{V_i} = [vx_i, vy_i, vz_i]^T$ are given. An error criterium between the dynamical system model and the observation is computed :

$$E = \sum_{i=1}^{i=N} \left\| \vec{V_i} - \left(A.\vec{X_i} + \vec{B} \right) \right\|$$

In order to use matrix forms the following matrices are introduced.

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_N \\ y_1 & y_2 & y_3 & \ldots & y_N \\ z_1 & z_2 & z_3 & \ldots & z_N \\ 1 & 1 & 1 & \ldots & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} vx_1 & vx_2 & vx_3 & \ldots & vx_N \\ vy_1 & vy_2 & vy_3 & \ldots & vy_N \\ vz_1 & vz_2 & vz_3 & \ldots & vz_N \end{bmatrix}$$

$$C = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$$

With such matrices, the error criterium E can be written :

$$E = \|V - C.X\|$$

To minimize E is the same as to minimize :

$$E^2 = \|V - C.X\|^2$$

The problem is to find the matrix C which minimize such a criterium. By using the matrix derivation properties, the gradient of E^2 is given by :

$$\nabla_C E^2 = -2.(V - C.X).X^T$$

The optimum is given by $\nabla_C E^2 = 0$

$$\Leftrightarrow C.X.X^T = VX^T$$

The optimum matrix C_{opt} is then given by :

$$C_{opt} = V.X^T.(X.X^T)^{-1}$$

The expression $X^T.(X.X^T)^{-1}$ is the pseudo inverse of the matrix X^T and can be writen as :

$$X^T.(X.X^T)^{-1} = L^T.S^{-1}.R$$

where S is the diagonal matrix of the singular values of the matrix X^T. This singular value decomposition is very helpful in order to avoid condition troubles. The matrix C is finally given by :

$$C = V.L^T.S^{-1}.R$$

The matrix A is then extracted from the matrix C, and the associated eigen value decomposition is given by :

$$A = L.D.U^T$$

The eigen values of the matrix D are complex numbers. In order to produce a scalar metric, the larger obsolute value of the real part of those complex eigen values is computed. The sign of this larger real part is related with the mode of the system (contraction or expansion). If those eigen values are drawn in the complex coordinates, it gives information about the level of contraction/expansion of the system and also its curl tendency. Such a metric can identify any speed vector organization (translation, curl or both) because it is sensitive to relative distances between aircraft. An example of computation of such a metric is given on figures 3,4,5. The traffic simulation consists in 41 aircraft involved in a fuzzy convergence 3. The evolution of real part of the larger eigen value is given on figure 4. As it can be noticed, the metric begins to be negative showing the situation is globally converging and after the crossing the metric becomes positive indicated the situation is now globally diverging. If such eigen value is represented in the complex coordinate system (see figure 5), the locus of the conjugate eigen value begins from the right side and move to the left side. The eigen value having an imaginary part means the system have a global curl moving during the crossing.

Linear system dynamical system modeling enable to produce an aggregate metric associated to any traffic situation and can recognize any global organization pattern. If detail metric is needed in order to indentify high (low) complexity areas, a non-linear dynamical system has to be used.

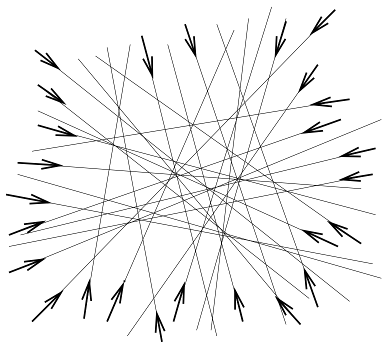

Figure 3: Fuzzy convergence of 41 aircraft

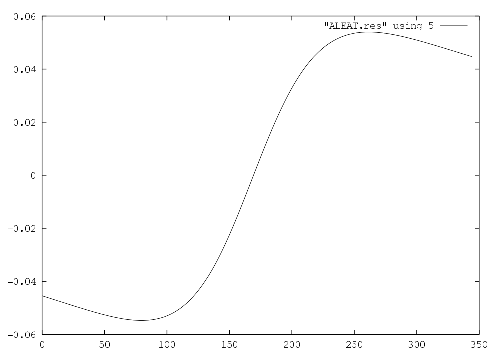

Figure 4: Evolution of the larger real part

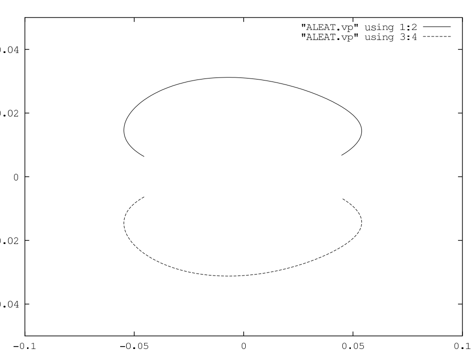

Figure 5: Evolution of the larger eigen value in the complex coordinate system

4 Non Linear extension

Interpolating splines are common tools in the field of numerical analysis. Those applications are optimal with respect to a cost functional that controls the smoothness of the solution and allows exact interpolation at given points [18]. One of the most elementary example of such family of applications is given by piecewise third order polynomials, named cubic splines, which give the smoothest interpolation with respect to the second derivative. Many generalizations of splines have been introduced, like vector splines [17] and splines on manifolds [8]. All those spline models may be used for static data, but dynamical system modelling requires time to be taken into account. Dynamical splines may be constructed as solutions to a optimal linear control problem. More specifically, let a controlled system :

$$\frac{dx}{dt} = Ax + bu \quad (1)$$

$$y = C^t x \quad (2)$$

with A, b, C matrices, x the state vector and u the command. The dynamic interpolation problem is to find the command u_0 that realizes the minimum of the functional :

$$\frac{\lambda}{2}\int_0^T u^2(t)dt + \frac{1}{2}\sum_{i=1}^{m}\left(y(t_i) - y_i\right)^t \left(y(t_i) - y_i\right)$$

where the $y_i, i = 1 \ldots m$ are the values to be taken by the system at times $t_i, i = 1 \ldots m$. The solution of this problem is known as smoothing spline and is computed as a linear combination of elementary kernel functions :

$$k_{t_i}(t) = \begin{cases} C^t \exp(A(t_i - t)) & t \leq t_i \\ 0 & t > t_i \end{cases}$$

This approach can be used to obtain smooth trajectories from samples.

4.1 Div-Curl Splines

Computing topological entropy for a given trafic situation requires interpolating a vector field given only samples (positions and speeds of aircraft at a given time). Vector spline interpolation seeks for the minimum of a functional of the form :

$$\frac{1}{2}\int_D \|LX(x)\|^2 dx + \frac{1}{2}\sum_{i=1}^m \|X(x_i) - V_i\|^2$$

where X is a vector field defined on a domain $D \subset \mathbb{R}^n$, L is an elliptic differential operator and $(x_i, V_i)_{i=1...m}$ are the interpolation data [7]. By introducing the adjoint operator L^t, optimal vector field can be shown to be a linear combination of shifted version of the elementary solution kernel of the PDE L^tL. A special case is the so-called "div-curl" splines with the criterion :

$$\int_{R^2} \alpha\|\nabla divX\|^2 + \beta\|\nabla curlX\|$$

with α, β positive reals. Relative values of these reals controls the smoothness of the approximation by focusing on constant divergence or constant curl.

4.2 Topological entropy computation

Combining trajectory extrapolation based on dynamic splines with vector spline interpolation yields a dynamic vector field model of the trafic suitable for evaluating topological entropy.

4.3 Vector dynamic splines

Mixing the previous approach, we will seek for an optimal solution of a dynamic interpolation problem, namely find a time-dependent vector field $X(t, x)$ defined on $[0, T] \times D$ and continuously differentiable up to order 2 in time coordinate that minimize :

$$\int_0^T \int_D \|\frac{\partial X(t,x)}{\partial t}\|^2 + \alpha\|LX(t,x)\|^2 dxdt$$

under the constraints :

$$X(t_i, x_i) = V_i,\ i = 1 \dots m$$

where α is a positive real. This value controls the relative importance of the vector field variation thru time over the discrepancy of X as measured by the differential operator L. Taking $\alpha = 0$ will yield to a constant vector field over time, while $\alpha \to +\infty$ will focus on the differential part.

4.4 Variational problem

Let L be an elliptic differential operator of order p with constant coefficients. L^t will denote the adjoint operator of L. We will assume in the following that the vector field X has fixed value at 0 and T and that for all $t \in [0, T]$, $X(t, .)$ belongs to the sobolev space H^p. Furthermore, the mapping $ttoX(t, .)$ must be a least two time continuously differentiable.

Let $\gamma : [0, T] \to C^p(R^n, R^n)$ a variation of X (that is time-dependent H^p vector field $\gamma(0, .) = \gamma(T, .) = 0$). Simple calculus of shows that the gradient of :

$$\int_0^T \int_D \|\frac{\partial X(t,x)}{\partial t}\|^2 + \alpha\|LX(t,x)\|^2 dxdt$$

with respect to γ is the linear mapping :

$$\gamma \to \int_0^T \int_D \langle\gamma, \frac{\partial^2 X}{\partial t^2} - \alpha L^t LX\rangle dxdt$$

The constraints on X impose that :

$$\gamma(t_i, .) = 0,\ i = 1 \dots m$$

One can find an elementary solution e of the PDE

$$\frac{\partial^2 X}{\partial t^2} - \alpha L^t LX$$

and use its translates e_{t_i,x_i} to represent the evaluation operator at (t_i, x_i). Combining this with the gradient previously obtained shows that the optimal solution is of the form :

$$X = \sum_{i=1}^m e_{t_i,x_i} + X_0$$

with X_0 an element of the kernel of the operator :

$$\frac{\partial^2 X}{\partial t^2} - \alpha L^t LX$$

5 Results

The topological entropy computation based on thin plate splines (equal importance of constant div and curl) and dynamic interpolation has been applied to the french trafic of the 11 August 2001 on some sectors. To speed up computations, an approximation of the true solution has been made with the help of a quasi-interpolation approach. The method can be sketched as follows. Since spline kernel is an elementary solution, applying the operator L^tL on it will yield to a delta distribution. By replacing in the differential operator all derivatives by finite differences approximations, the resulting operator applied on the kernel will yield a bell-shaped function that approximate the delta distribution. Those functions are used to produce an interpolating vector field without having to solve a linear system for coefficients. The following figures show examples of computation of such a metric for two sectors (Z1, JS). Two kind of simulation are presented : the first one with standard routes and the second one with direct routes. The absciss represents times samples (each ten second for a full day). Sector JS is more complicated than Z1. Based on those results it can be concluded that direct route assignment will decrease the complexity of JS, but will increase for Z1 (note however that complexity peaks are more concentrated in time for direct routes). The map of french sector is given at the end of the article.

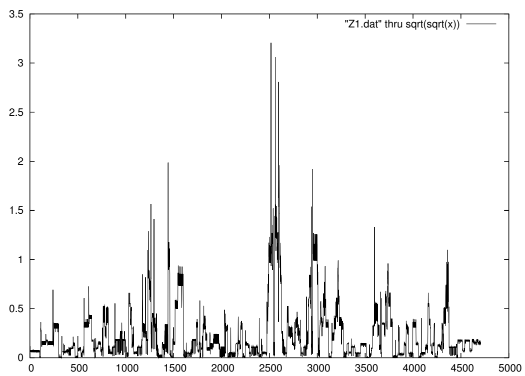

Figure 6: Z1 sector

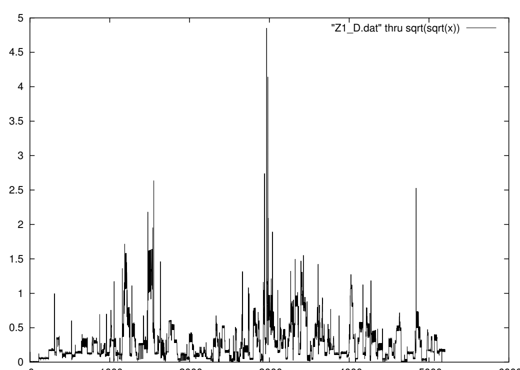

Figure 7: Z1 sector direct routes

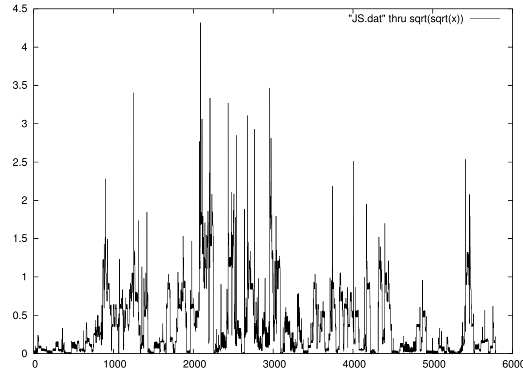

Figure 8: JS

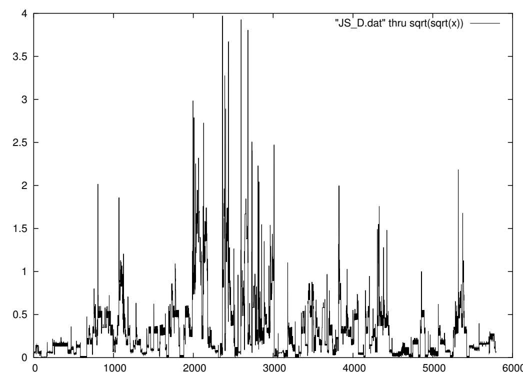

Figure 9: JS sector direct routes

6 Conclusion

We have presented in this paper a new air traffic complexity metric based on non-linear vector field model of air traffic. Extending previous results on topological entropy, this method allows identification of traffic pattern organization in its full generality while previous works were limited to specific aspects of the complexity. Furthermore, since we are processing trajectory segments instead of samples at a given time, the induce metric is more realistic from an operational point of view. The quasi-interpolation algorithm allows real-time processing on operational traffic even for large areas (Europe or US). Unlike linear models which produce mean complexity indicators, the non-linear one may give local information, thus providing a way of displaying maps of complexity. In a future work, such a tool will be applied to comparison of US and Europe airspace.

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7 biography

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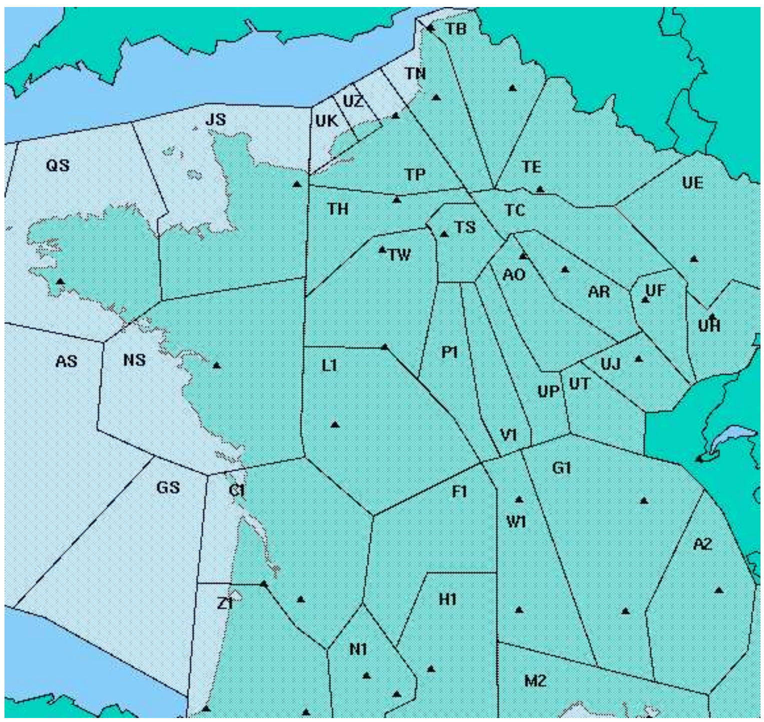

Figure 10: French airspace