# DEVELOPMENT OF AN ALGORITHM TO MODEL THE LANDING OPERATIONS, BASED ON STATISTICAL ANALYSIS OF ACTUAL DATA SURVEY (proESTOP)

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### Abstract

This paper presents the overall method followed to develop the stochastic algorithm based on Montecarlo simulations used by PROESTOP, a tool that estimates the runway occupancy time for arrivals (ROTA) and provides new exit taxiways optimum allocation on a given scenario. The tool, developed by INECO, is currently in use in different projects, namely the development and revision of Spanish airports master plans, the SPADE project within the VI EU Framework Programme and more recently, after the acquisition of a license, in support to studies carried out by Eurocontrol.

This paper describes the whole process from the review of existing methods to the final validation of the model: Reasons for actual data need, and existing models deficiencies are mentioned and illustrated. Data survey design, execution and post-process is presented, including estimation of errors, outliers identification and fitting regression models. Discussion of most influence variables on ROTA, based on statistical analysis, is summarised. An alternative clustering of aircraft models attending their performances is introduced. An Estimated Logistic regression model is described to estimate the exit taxiway Different alternative probability. speed functions along the runway are presented and compared with actual landing data. The steps required to deal with a stochastic model based on Montecarlo simulations are described. The use of standard (Normal, Erlang, etc.) and nonstandard (Johnson curves) probability distribution for fitting the input variables was introduced. Description and outcomes of the validation process are presented. And, finally, the current applications of the tool are mentioned.

## Introduction

INECO, the leading Spanish consultancy and engineering firm specialised in the transport

industry, has developed, using internal R&D funding, a tool for the assessment of runway occupancy time and runway exit taxiway allocation (PROESTOP). The objective of the project was to develop a friendly, flexible and reliable tool for evaluating and proposing optimal allocation of new exit taxiways on a given scenario.

The main factor to be considered when assessing new exit taxiways is runway occupancy time on arrival (ROTA). Therefore, a model for estimating ROTA was developed.

The overall project was broken down into five phases: review of existing methods, actual data survey, statistical analysis, modelisation and validation. A brief summary on these phases as well as the description of the resulting software tool are presented in this paper

# **Background. Review of existing** methods

#### Need for an actual data survey.

ROTA is defined as the time elapsed since the landing aircraft flies over the threshold to the moment when the plane vacates the runway by an exit taxiway [1][9]. Typically, methods for estimating ROTA are based on kinematic formulae, simulations or local ROTA surveys. As in any other engineering problem, a trade-off between precision and required effort must exists.

Whilst data collection ensures the most accurate estimates of ROTA for the local conditions (provided a sufficient sample size), it also entails the highest costs in time and resources. Furthermore, depending on the local conditions the actual ROTA for the current situation might not be enough to analyse what-if scenarios (for instance, the impact of a new exit taxiway might not be extrapolated from existing ones). Examples of different data acquisition methodologies designed by Eurocontrol, Aena, NATS or LVNL can be found in [1]. The alternative are the analytical methods, which are more economical and simple options. The most popular ones are the kinematic formulae (i.e., assuming a constant deceleration profile) [7] or the Runway Exit Design Interactive Model (REDIM v2.1), a simulation model carried out for the Federal Aviation Administration (FAA) as part of a research grant with Virginia Tech [14].

The kinematic formulae base their analysis on an average performance of the aircraft. But daily experience proves that actual ROTA shows a great dispersion, which affect on the predictability of the operation and therefore on the decisions that pilots and controllers must take.

On the contrary, REDIM applies an stochastic model to deal with uncertainty. But deceleration and turnoff speed, which in the "real world" are decided on a case by case basis by the pilots, are defined as input data and, therefore the user must know it in advance to introduce it as a given constant value. The model has been used in Spanish airports, but a case by case calibration with actual data was required [9].

Therefore, an actual data survey was designed to obtain information about the factors affecting landing operation. The final objective was to develop a model to reproduce the aircraft performance on the runway, taking into account the variability of both input and output data.

## Actual data survey

#### Design and preparation of the survey

The data acquisition consisted on the recording of the time over designed intermediate points along the runway, based on visual observations from the tower of control.

The selected scenario was runway 33 at Madrid-Barajas airport (Figure 1). At the time of the survey four rapid exit taxiways (RETs) and three right-angle exits were available on the runway.

The intermediate points, between the threshold and the exit taxiways, were selected so that a clear visual reference could be established. The landing distance for each point is computed as the intersection of the sight line from the tower and the runway centreline (Table 1).

Some of these points are exit taxiways (#7-#14). So the number of intermediate points available for each flight varies from 8 up to 14 depending on the turn off point. Finally, touchdown is defined as an extra event for recording. As no distance measurement is feasible during the data acquisition, only the time of the event is recorded.

The parallax error was computed for each point to verify that an adequate precision could be achieved by this procedure.



Figure 1. Data acquisition layout

Given the perpendicular distance from the observer to the runway centreline, and the angle to the reference point, the error of the visual observation at a point x  $(dT_x)$  is given by:

$$dT = \frac{dS}{V}$$
, at x

Where,

$$S = D \cdot tg\theta$$
$$dS = (D \cdot \sec^2 \theta) \cdot d\theta$$

Thus



Figure 2. Parallax error

Given the actual distance from the tower (650m) and provided a typical landing speed at each point (from 60 m/s on threshold to 5 m/s on vacating) and a typical angle resolution of the human eye ( $d\theta$ =1 minute of arc) [2], the parallax error is estimated at each intermediate point.

As later confirmed, it was observed that even in the worst case (aircraft vacating by the last exit) the visual error (0.3 seconds) would be less than the sampling error (0.4 seconds) (Table 1). Hence it was assumed that the visual error was included in the overall sampling error [3].

#### Data acquisition process

The survey was performed for three weeks preferably during the peak hours of traffic, not only to obtain more data but also to ensure high demand conditions. Besides the runway occupancy times, the flight number, the exit used and the meteorological information (visibility, cloud ceiling, wind speed and direction and dry or wet runway) were recorded.

On a first stage (1000 flights), all arrivals were recorded. Then the sample was analysed so that representative models with less available data were identified. From this point on, the survey hours were selected according to the scheduling hours of these models. The survey was completed when 1500 data were recorded. No landing on wet runway was recorded. Neither light traffic, as Barajas is closed to General Aviation and Business aircraft [4].

As cited before, the overall sampling error was 0.4 seconds with a significance level of 95%.

| Sample<br>size | Mean<br>ROTA | Standard<br>deviation<br>ROTA | Sampling<br>error<br>ROTA |  |
|----------------|--------------|-------------------------------|---------------------------|--|
| 1500           | 46.5 s       | 7.9 s                         | ±0.4 s                    |  |

# Table 1. Summary statistics of the data sample (significance level 95%)

#### Data post processing

A first post-process allowed to complete the required information for each flight: airline, aircraft model, origin airport and stand. The actual landing weight was approximated by a categorical variable based on the duration of the flight and the number of passengers. The postprocess also allowed to estimate the touchdown distance to threshold by interpolating between the distances of the points immediately before and after the touchdown (sorted by time).

In parallel, a complete database with the performance and dimensions of the most representative models was generated (136 records), as well as a complete list of 2655 aircraft designators with their equivalent aircraft performance.

Finally a complete table was produced with the flight information (airline, origin, stand), the aircraft characteristics (model, weights, reference speed, dimensions), the meteorological data (tailwind and crosswind speeds, visibility, cloud ceiling and runway condition (dry or wet)) and the runway performance (overall ROTA, intermediate times, location and type (rapid or right-angle exit) of the exit taxiway and average speeds along the runway).

### **Statistical analysis**

The objective of the statistical analysis was to identify the main factors affecting the runway performance and the relationship between them. Analysis of variance tests (ANOVA) were carried out to check whether the differences between the populations means were significant.

Prior to the ANOVA tests, outliers (points more than three inter-quartile ranges below the first quartile or above the third quartile) were identified and eliminated from the sample (19 records).

The available factors can be grouped in: kinematic factors, aircraft performances and runway and environmental characteristics.

Table 7 shows the dependencies between the factors analysed.

# Discussion of the dependency between factors

#### Kinematic factors

They represent the output variables to be modelled:

| Variable               | Description                         |  |  |  |  |  |
|------------------------|-------------------------------------|--|--|--|--|--|
| V                      | Speed over threshold (ground        |  |  |  |  |  |
| * THR                  | speed)                              |  |  |  |  |  |
| V <sub>EAS</sub>       | Equivalent air speed over threshold |  |  |  |  |  |
| V                      | Average speed in the last link      |  |  |  |  |  |
| V COASTING             | before the exit taxiway.            |  |  |  |  |  |
| V <sub>EXIT</sub>      | Turn off speed                      |  |  |  |  |  |
| a <sub>MEAN</sub>      | Mean deceleration                   |  |  |  |  |  |
| a <sub>EXIT</sub>      | Deceleration during turn off        |  |  |  |  |  |
| v                      | Distance from threshold to          |  |  |  |  |  |
| $\Lambda_{\text{TD}}$  | touchdown                           |  |  |  |  |  |
| X <sub>EXIT</sub>      | Distance to exit taxiway            |  |  |  |  |  |
| ROTA                   | Runway occupancy time               |  |  |  |  |  |
|                        | Type (Angle) of the exit taxiway.   |  |  |  |  |  |
| $\Phi_{\mathrm{EXIT}}$ | Rapid exit (30°) or Right-angle     |  |  |  |  |  |
|                        | (90°).                              |  |  |  |  |  |

Table 2. Kinematic variables

According to the ANOVA,  $V_{THR}$  affects both  $X_{TD}$  and  $X_{EXIT}$ . It is logical since the faster over threshold, the longer distance is required to decelerate. Besides, pilots may use flare to control the speed, shortening or lengthening the touchdown [5].

On the other hand,  $V_{EAS}$  only affects  $V_{THR}$  as the latter is the translation to ground speed of the former. This seems counterintuitive, but it is due to the effect of the wind. Given two aircraft with similar  $V_{EAS}$ , but different wind conditions, they will land with different  $V_{THR}$  which, as seen before, may lead to different landing distances and ROTs.

 $X_{TD}$  only affects  $V_{THR}$ . There is no relationship with decelerations, ROT or  $X_{EXIT}$ , which may be explained by the aforesaid speed control during flare.

The exit used has been grouped in acuteangle exits or rapid exit taxiways (RET) and right-angle exits. As expected, the landing distance depends on the rest of the factors, excepting  $V_{EXIT}$  when vacating by right-angle exits. That means that aircraft vacating by a right-angle exit have previously reduced their speed down to similar (low) values, independently of the selected exit.

#### Aircraft performances

Aircraft characteristic as defined by constructor:

| Variable         | Description                                 |  |  |  |  |  |  |
|------------------|---|--|--|--|--|--|--|
| ID               | Categorical variable representing           |  |  |  |  |  |  |
| <b>ID</b> PERF   | the equivalent performance model            |  |  |  |  |  |  |
| V <sub>REF</sub> | Reference approach speed                    |  |  |  |  |  |  |
|                  | 1.3·V <sub>STALL</sub> (Indicated airspeed) |  |  |  |  |  |  |
| MLW              | Maximum Landing Weight                      |  |  |  |  |  |  |
| LD               | Landing distance to full stop               |  |  |  |  |  |  |

#### Table 3. Performances variables

As expected, ANOVA shows a clear dependency between aircraft performances and output variables.

# Runway and environmental characteristics

The runway and environmental characteristic analysed are:

| Variable           | Description   |
|--------------------|---|
| F <sub>v</sub>     | Dimensionless speed: V <sub>EAS</sub> /V <sub>REF</sub>                           |
| VTAILWIND          | Tail wind component of the speed  |
| Fw                 | Dimensionless weight: LW / MLW  |
| STAND              | Categorical variable representing<br>the apron area where the stand is<br>located |
| ID <sub>PERF</sub> | Categorical variable representing the equivalent performance model                |
| AIRLINE            | Categorical variable that   |

| identifies airlines Spanish (familiar  |
|--|
| with the airport) and foreign airlines |
| (more likely unfamiliar).              |

#### Table 4. Runway variables

 $F_v$  affects ROTA,  $X_{\text{EXIT}}$  and  $\Phi_{\text{EXIT}}$  through  $V_{\text{THR}}.$ 

Wind affects all the kinematic variables except ROTA,  $\Phi_{EXIT}$  and  $a_{MEAN}$ . That is,  $V_{TAILWIND}$  has an impact on landing distance, but not on the overall ROTA, as also found in [6]. This seems to confirm that the landing distance depends on the ground speed over threshold.

Weight affects all the variables except  $\Phi_{\text{EXIT}}$  and  $a_{\text{EXIT}}$ . It seems logical that weight affects the speed profile along the runway, but not necessarily the type of exit used. But it also shows that pilots applies different deceleration rates and adjust  $V_{\text{EXIT}}$  and  $a_{\text{EXIT}}$ , once an exit is selected.

The analysis of the airline and stand factors confirms that, as initially suspected, the familiarity with the airport and the stand location may impose conditions on the pilot's decisions.

#### **Outcomes**

The goal of the statistical analysis was to identify the variables that should be taken into account to model the runway occupancy time (ROTA) and the exit taxiway selection ( $X_{EXIT}$ ,  $\Phi_{EXIT}$ ): Aircraft characteristic and performances,  $F_V$ , Wind, deceleration,  $F_W$ , Stand location and familiarity with the airport.

Some qualitative conclusions were:

· Wind does affect the landing distance, but not ROTA directly. Therefore, a better variable to model speed over threshold should be  $F_{V}$ . On the contrary,  $V_{TAILWIND}$  has an impact on the exit taxiway selection.

• Touchdown distance depends on pilot techniques and familiarity with the layout, but has not a significant impact on ROTA.

 $\cdot$  Pilots adjust the speed profile when vacating by a given exit. Therefore deceleration rate and V<sub>EXIT</sub> depend on the exit location.

• Pilot's overall behaviour are affected by other operational factors such as the familiarity with the airport and the stand location. Therefore, clear indication on exit locations and design is a key factor to enhance the runway performance, as stated in [1] and [10]. • As a consequence of the non-linearity of inputs (kinematic variables) and outputs (turn off distances and ROTs) symmetry is broken, what leads to non-correspondent values for mode inputs (even with multimode output distributions) and unexpected behaviours (ROTA outliers).

For example Figure 3 shows the box-and-whisker plot of ROTA and  $V_{EAS}$ . It can be observed that meanwhile the speed distribution is clearly symmetric (standard skewness - 0.999), the ROTA distribution has a clear upper tail (standard skewness 12.849).



Figure 3. Comparison of ROTA and V<sub>EAS</sub> distributions

## **Experimental model design**

#### Model Overview

The conclusions of the statistical analysis confirm that a stochastic approach should be preferred to a deterministic one. Considering the probabilistic distributions of both input and output data allows to assess the deviations from the expected or most common behaviours, enhancing the robustness of the outputs.

A usual way of dealing with stochastic models is using Montecarlo methods. The overall method can be summarised as:

1) To build the algorithm: actual data sample is analysed so as to construct an algorithm to estimate both the ROTA and the exit usage.

2) To model the input variables: input variables are identified and fitted to probabilistic distributions in accordance with actual data sample.

3) To run the simulations: the model generates pseudo-random numbers by simulation techniques to mimic the statistical distribution of the input variables. Then, it obtains the correspondent output variables by computing the algorithm. After a sufficient number of replications, the sampling results will reproduce the distribution of the statistic outputs.

#### Simplifications

One of the most important user requirements in the design of the model was the simplicity of the user-inputs. Therefore, some simplifications were required. Some of the influence variables identified in the survey were not finally included in the model, due to the difficulty to estimate them in a generic scenario:

 $\cdot$  F<sub>W</sub> (Weight factor): which depends on the origin of the traffic.

• Airline Factor: which depends on the familiarity of the pilots with the airport layout.

• Wind: whether no wind information is available, clam wind conditions will be supposed.

#### **Aircraft classification**

To estimate the distribution of the kinematic variables for each aircraft type a significant amount of data is required. Thereby, records from the Barajas survey were grouped in clusters with similar aircraft characteristics and runway performance.

Instead of using directly the usual classifications according to  $V_{APP}$  (A-D) or wake vortex category (H, M, Lt), a cluster analysis was performed using the K-Means method<sup>1</sup>.

Several clustering were performed trying with different numbers of clusters (from 4 to 8). The final number of clusters (7) was obtained as a trade-off between the uniformity of the aircraft within each cluster and the amount of data available in each cluster.

The aircraft model distribution within each cluster was analysed (Table 5). It was observed that the clusters were conformed not only around the wake vortex category (MTOW criteria) and the threshold speed, but they also fitted reasonably well with the constructor (Table 6).

| Cluster | Н             | Х   | М   | Α   | В   | 0   | Т   |
|---------|---------------|-----|-----|-----|-----|-----|-----|
| #1      | 0%            | 0%  | 28% | 27% | 44% | 1%  | 0%  |
| #2      | 1 <b>00</b> % | 0%  | 0%  | 0%  | 0%  | 0%  | 0%  |
| #3      | 0%            | 0%  | 0%  | 0%  | 1%  | 75% | 24% |
| #4      | 21%           | 62% | 0%  | 7%  | 10% | 0%  | 0%  |
| #5      | 0%            | 0%  | 0%  | 1%  | 0%  | 10% | 89% |
| #6      | 0%            | 0%  | 34% | 17% | 47% | 1%  | 0%  |
| #7      | 12%           | 12% | 19% | 25% | 28% | 4%  | 0%  |

# Table 5. Percentage of aircraft type within each cluster

<sup>&</sup>lt;sup>1</sup> The K-Means method is a non-hierarchical method; each cluster begins with a specified number of groups, each of which has a single random point. A sequence of points is sampled, and each point is added, in turn, to the group whose mean it is closest to. The group mean is then adjusted.

Then for simplification and based on the distribution of traffic within each cluster, an alternative classification was adopted, attending ICAO wake vortex category, engine type and constructor.

| Aircraft Group            | Acronym |
|---------------------------|---------|
| Heavy                     | Н       |
| B757                      | Х       |
| MD80s series (Medium Jet) | М       |
| Airbus Medium Jet         | А       |
| Boeing Medium Jet         | В       |
| Other Medium Jet          | 0       |
| Medium Turboprop          | Т       |
| Light                     | Lt      |

#### Table 6. Aircraft group

Light traffic, although not present in the survey, was included as a different group apart from the rest.

#### Logic of the model

In order to simplify the problem, the runway performance model is split in two complementary models:

1) a model to estimate the accumulated probability of using each exit taxiway and

a kinematic model based 2) on dimensionless distance variable which estimates  $V(\xi)$  for each type of exit (right-angle, RET) and aircraft group, where  $\xi = \frac{x}{X_{\text{EXIT}}}$ .

By doing this, a more simple approach of the kinematic profile along the runway can be achieved, reducing the influence of the exit location in the speed function along the runway.

Figure 4 shows the overall logic of the model. As stated before, the model applies Montecarlo methods so as to obtain the ROTA and exit taxiway.

For a given aircraft and given meteorological conditions, the model randomly assigns the runway exit and the kinematic variables according to the correspondent probability distributions of the aircraft's group. Then ROTA is calculated by numerical integration of the speed function.

The model includes a final check, so that excessive decelerations are prohibited (namely, 2 m/s<sup>2</sup> on dry runway and 1.5 m/s<sup>2</sup> on wet runway) [7] [8]. When an acceptable deceleration is not possible after several iterations, the model tries with the next available exit.

To obtain statistically significant results on ROTA a sufficient number of flights must be simulated. The higher the number, the better the simulated output distribution fits with the actual distribution. The minimum number varies in each case, depending on the desired precision (admissible error: e, z-score for the given significance level  $\alpha$ :  $Z_{\alpha}$  ) and the ROTA distribution (standard deviation of the output:



Figure 4. Model overview

For example, in a typical scenario with ROTA mean 60 seconds and standard deviation seconds, the minimum number of 10 replications should be 2,189, provided admissible error of 1% and a significance level of  $99.5\%^2$ .

#### Turn-off distance probability model

The probability density function of the exits can be computed as the difference between the accumulated probabilities of the precedent and the given exit. By using the cumulative distribution function (CDF), instead of the density function, different airport layouts can be compared, regardless the number and location of their exit taxiways.

Empirical CDF can be found in [10] and [11]. Although they provide a good guidance on runway exit usage for different aircraft groups, they lack to take into account important factors,

<sup>&</sup>lt;sup>2</sup> Note that this would also be the minimum sample size required to obtain a similar sampling error.

such as elevation or temperature ([11]) or rightangle exits ([10]).

The proposed model is based on an Estimated Logistic regression model (obtained by the Maximum likelihood method) of actual exit taxiway usage. Statistics of the Spanish airports was compiled from [9]. It comprises 23 different airports and more than 22.000 landing operations.

The Estimated Logistic regression fits a response of observed proportions or probabilities at each level of an independent variable. The response function is a non-linear S-shaped curve with asymptotes at 0 and 1. The dependent variable must have values greater than 0 and less than 1. The model is defined by the equation:



Figure 5. Example of the Estimated Logistic regression

An initial comprehensive set of parameters was included in the model accounting for geometrical (runway length, exit location and geometry, number of exit, apron relative location, slope, etc.) and meteorological factors (elevation, temperature, mean tailwind speed).

The model was simplified removing the variables with highest p-value until the highest order term was statistically significant. The adjusted percentage of deviance explained by the model ( $\mathbb{R}^2$ ) for the different aircraft groups was higher than 0.9 for each of them.

In order to ensure the operational validity of the output, the value obtained from the regression is corrected by some heuristics. For example, the accumulated probability of the last exit shall always be 1, the probability of using a given exit by bigger aircraft will be as high as for the smaller ones (i.e., smaller aircraft will likely vacate before the bigger ones), and so on.

#### Runway speed profile model

The objective is to model the landing speed function along the runway from the threshold to a given exit taxiway for each aircraft group.

A first attempt was made assuming constant deceleration (a):

$$V(x) = V_{THR} - a \cdot t$$
$$x = V_{THR} \cdot t - \frac{1}{2}a \cdot t^{2}$$

Where the exit speed by a given exit distance  $(X_{EXIT})$  is

$$V_{EXIT} = \sqrt{V_{THR}^2 - 2 \cdot a \cdot X_{EXIT}}$$

This option is logical as the calculation of the coefficients from survey data is very simple and it is the most common assumption in ROTA estimations [7]. The coefficients, initial speed ( $V_{THR}$ ) and deceleration rate (a), are obtained by a linear least squares regression of the Distance-Time paired data from the survey.

Even though the regressions provide extremely good adjustments of distance and time ( $R^2 \sim 0.9$ ) the resulting speed functions are not satisfactory. Constant deceleration imposes braking action earlier than normal in real operation and speeds over threshold much higher than observed.

The same conclusions can also be found in [8], where an alternative non-linear speed function was proposed:

$$V = V_{\text{THR}} - (V_{\text{THR}} - V_{\text{EXIT}}) \cdot \xi \cdot e^{-K(1-\xi)}$$

where  $\xi = \frac{x}{X_{\text{EXIT}}}$  is a dimensionless

distance variable.

The idea behind this function is to decelerate less initially and more later. Pilots may follow the non-linear profiles as heavier deceleration is delayed [8].

Figure 6 shows the comparison of an example of the non-linear and the constant deceleration functions adjusted to an actual flight from the Barajas survey. The different deceleration rate, specially during flare, can be observed. In the example, the constant deceleration model imposes reductions of speed of more than double a conservative value of 5Kt, suggested in [7].

In consequence, the estimated ROTA based on the constant deceleration model is in general too optimistic. In the example the adjusted model for a flight from the Barajas survey yields a ROTA of 36.9 seconds, versus an actual ROTA of 38.3 seconds (-3.6%).



Figure 6. Comparison of an example of the non-linear speed and the constant deceleration functions

On the contrary, the non-linear model fits better the usual aircraft landing performances and thus provides a better adjustment of ROTA. In the example, the estimation of ROTA matches the actual ROTA.



# Figure 7. Speed coefficients estimation process

The adjustment of the non-linear function depends on three parameters: initial speed  $(V_{THR})$ , vacating speed  $(V_{EXIT})$  and a deceleration constant (K), which actually controls the delay in the beginning of the braking [8].

Intermediate speed versus dimensionless distance records from Barajas survey were

adjusted to this non-linear function. Figure 7 shows the process followed for each flight.

The method consisted on the iterative estimation of the parameters, by minimising the sum of squares of the residuals while ensuring that estimated ROTA matches the actual one. An initial value of  $F_V=1$  was used and then a least square regression of the linearised speed function was obtained. The parameters of the function ( $V_{THR}$ ,  $V_{EXIT}$  and K) could be then calculated for the given  $F_V$ . When the estimated ROTA did not match actual ROTA a higher  $F_V$  was tested.

 $F_V$  was used instead of directly the initial ground speed because, as stated in the outcomes of the statistical analysis of the survey,  $F_V$  is more significant to landing performance (ROTA and  $X_{EXIT}$ ) than  $V_{THR}$  (ground speed).

The last step was to fit the resulting distributions of the parameters ( $F_V$ ,  $V_{EXIT}$  and K) to probabilistic distributions to be used in the stochastic model.

Standard distributions were tested (namely Normal, Erlang, Lognormal, Weibull, Uniform, Gamma and Poisson) and selected according to a series of goodness-of-fit tests and p-values based on the empirical distribution function (EDF)<sup>3</sup>. When neither of the standard distributions fitted the empirical sample, a set of Johnson curves were tested<sup>4</sup>.

The advantage of this approach is that once a particular Johnson curve has been fit, the Normal distribution can be used to obtain the pseudo-random numbers instead of working with an empirical distribution. Methods for fitting Johnson curves, so as to approximate the first four moments of an empirical distribution, are described in detail in [15], and [16].

Finally, the independence of the resulting distributions (Fv,  $V_{EXIT}$ , K) for each type of exit and aircraft group was checked. Their independence allows the model, during Montecarlo simulation, to randomly select each of these variables regardless the others (Figure 4).

distributions).

 <sup>&</sup>lt;sup>3</sup> The EDF tests (Kolmogorov-Smirnov, Chi-Square, and Anderson-Darling) are based on various measures of discrepancy between the empirical distribution function and the cumulative distribution function based on the specified distribution.
 <sup>4</sup> Johnson Curves. Johnson (1949) described a system of frequency curves that represents transformations of the standard normal curve (see [15], for details). By applying these transformations to a standard normal variable, a wide variety of non-normal distributions can be approximated, including distributions that are bounded on either one or both sides (e.g., U- shaped

## Validation of the model

#### Validation process

The objective of the validation was to verify the fulfilment of all the requirements, so that the tool complied user needs.

The validation fell within the Functionality Testing category. It verified whether the functionality defined in the System Requirement Analysis Report were available. From this point of view, the system was like a black box. No knowledge about the internal structure of the system was needed. The tests were based on the expected requirements and the inputs/outputs of the tool.

The validation tests were carried out by the "user", based on actual data and in the usual working environment, that is, with the software, hardware and users which the tool has been designed for. In this case, the "user" was a task group, apart from the design team, formed 'ad hoc' within INECO to perform the tests. <sup>5</sup>

#### **Output requirements**

The validation of output requirements consisted on a test battery comprising different reference scenarios. Some quantitative indicators obtained from PROESTOP were compared with those obtained from actual operational data and REDIM tool [14]. The objective was to bound the level of reliability of the outputs.

#### Performance benchmark

The chosen performance benchmark indicators were:

**Exit Taxiways Usage Percentage**. The p-value of the Chi-Square test was chosen as the performance benchmark indicator for this parameter. Particularly, the significance of the deviations from the sample p% is used, where the expected value ( $e_j$ ) comes from the actual data. The benchmark yields between 0% and 100%, where 0% means a complete coincidence.

$$\chi^{2} = \sum_{j=1}^{K} \frac{(o_{j} - e_{j})^{2}}{e_{j}}$$

• **Mean Runway Occupancy Time**. The relative error is used as the performance benchmark indicator for this parameter.

#### **Test-bed airports**

Five Test-Bed Airports were selected so as to include the most representative typologies: three or more RETs, 1 RET, no RET, no Parallel taxiway.

By doing this, PROESTOP was tested under different physical and traffic conditions. A complete summary of results can be found in Table 8.

#### Validation outcomes

The main validation outcome was the acceptable validity of estimations: Less than 5% error in overall mean ROTA and no significant differences in exits usage (95% confidence).

When actual data sample has a low statistical significance (small sample and/or a high deviation), PROESTOP estimation is even within the confidence interval of the sample mean. On the contrary, estimates by REDIM are too low. As observed in [9], although the model has been validated in the USA, its assumptions seem optimistic for European airports.

# The runway occupancy time estimator tool (PROESTOP)

Parallel to the construction of the algorithm, a user's software interface for Windows platforms was developed to enable a flexible and friendly use of the algorithm, and a straightforward integration with other tools thanks to its modular architecture and the extensive use of standard formats (txt, mdb, html and xml format files).

The current release (v4.0) provides estimates of runway occupancy time statistics and optimal proposals for the allocation of new exit taxiways given the runway layout, the fleet and the meteorological conditions. It includes several enhancements including the estimation of runway occupancy times on departure (ROTD), and the validation of results for wet runways.

To date, PROESTOP has been used in the development and revision of ten Spanish airports master plans and has been successfully integrated in the Supporting Platform for Airport Decision-Making and Efficiency Analysis project (SPADE) within the VI EU Framework Programme [12].

Recently, the Airports & Environment Management Business Division (DAS/AEM) of

<sup>&</sup>lt;sup>5</sup> Prior to the validation, a similar process to the described here was specifically performed to model the light traffic. So validation outputs presented here, includes the light traffic [13].

Eurocontrol has acquired a license of PROESTOP 4.0 to support its studies.

# Conclusions

The statistical analysis has confirmed the need of an stochastic approach for modelling runway performance. The variability of input and output data affects the robustness of the conclusions extracted from the model, when evaluating the existing infrastructures or proposing new exit allocations.

Furthermore, validation has proved that proESTOP is a valid tool for a cost-effective estimate of the ROT in those scenarios when the required accuracy is within the margin of error provided by the tool. For example, in those cases where ROT is not the most limiting factor to establish separations, where the sampling error is likely to be high (because an insufficient sample is expected due to budget or time limitations - e.g. low demanded airports), or simply because data sampling is not feasible at all (what-if analysis).

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# Key words

Runway Occupancy Time, exit taxiway, Montecarlo simulation, modelling, statistical data processing, software implementation.

# **Biographies**

**Carlos Barbas**, project leader of PROESTOP, joined INECO in 1998 and has worked in several ATM simulation projects, including Aena's runway capacity model (PICAP+). He has a degree in Aeronautical Engineering from the Polytechnic University of Madrid.

**Javier Vázquez** Aeronautical Engineer from the Polytechnic University of Madrid, joined INECO in 1998, took part in the development of the PICAP runway capacity programme as well as other ATM studies on airport operations.

**Pablo López** was part of the proESTOP development team. He joined INECO in 2003 and has worked in several projects as system engineer. Currently he is analyzing the ATC system complexity. He has a degree in Aeronautical Engineering from the Polytechnic University of Madrid.

# Appendix

|                       | Variable |                   |                      |                  |           |                   |                          |                   |                 |
|-----------------------|----------|-------------------|----------------------|------------------|-----------|-------------------|--------------------------|-------------------|-----------------|
|                       | ROTA     | X <sub>EXIT</sub> | $\Phi_{\text{EXIT}}$ | V <sub>THR</sub> | VCOASTING | V <sub>EXIT</sub> | <b>a</b> <sub>MEAN</sub> | a <sub>EXIT</sub> | X <sub>TD</sub> |
| Factor                |          |                   |                      |                  |           |                   |                          |                   |                 |
| V <sub>REF</sub>      | <0.0001  | <0.0001           | 0.0004               | <0.0001          | <0.0001   | <0.0001           | <0.0001                  | <0.0001           | <0.0001         |
| MLW                   | <0.0001  | <0.0001           | <0.0001              | <0.0001          | <0.0001   | <0.0001           | <0.0001                  | <0.0001           | <0.0001         |
| LD                    | <0.0001  | <0.0001           | <0.0001              | <0.0001          | <0.0001   | <0.0001           | <0.0001                  | <0.0001           | <0.0001         |
| ID <sub>PERF</sub>    | <0.0001  | <0.0001           | <0.0001              | <0.0001          | <0.0001   | <0.0001           | <0.0001                  | <0.0001           | <0.0001         |
|                       |          |                   |                      |                  |           |                   |                          |                   |                 |
| VTAILWIND             | 0.1740   | 0.0044            | 0.1979               | <0.0001          | 0.0047    | 0.0003            | 0.2264                   | 0.0401            | 0.0608          |
| $F_{W}$               | <0.0001  | <0.0001           | 0.9789               | <0.0001          | <0.0001   | 0.0002            | <0.0001                  | 0.1345            | <0.0001         |
| STAND                 | <0.0001  | <0.0001           | <0.0001              | <0.0001          | 0.0178    | 0.0096            | 0.1136                   | <0.0001           | 0.9260          |
| AIRLINE               | <0.0001  | 0.0441            | <0.0001              | 0.0041           | 0.0129    | <0.0001           | 0.2095                   | <0.0001           | 0.0025          |
| F <sub>v</sub>        | <0.0001  | 0.0001            | <0.0001              | <0.0001          | 0.0736    | 0.0584            | 0.1261                   | 0.0592            | 0.4213          |
|                       |          |                   |                      |                  |           |                   |                          |                   |                 |
| V <sub>EAS</sub>      | 0.8717   | 0.1459            | 0.9876               | <0.0001          | 0.2031    | 0.3644            | 0.1823                   | 0.5013            | 0.7073          |
| V <sub>THR</sub>      | 0.9788   | <0.0001           | 0.0695               | N/A              | 0.9362    | 0.1317            | 0.9442                   | 0.9944            | 0.0315          |
| X <sub>TD</sub>       | 0.8581   | 0.1790            | 0.4990               | 0.0064           | 0.2849    | 0.4471            | 0.0085                   | 0.7310            | N/A             |
| X <sub>EXIT</sub> 30° | <0.0001  | N/A               | <0.0001              | <0.0001          | <0.0001   | <0.0001           | <0.0001                  | <0.0001           | <0.0001         |
| X <sub>EXIT</sub> 90° | 0.0002   | N/A               | <0.0001              | <0.0001          | <0.0001   | 0.4362            | 0.0213                   | 0.0215            | 0.0418          |
| $\Phi_{\text{EXIT}}$  | <0.0001  | <0.0001           | N/A                  | 0.0002           | 0.1532    | 0.0007            | 0.0004                   | <0.0001           | <0.0001         |

 Table 7. Dependency between factors<sup>6</sup>

| Test bed<br>runway | Elevation | Exit<br>taxiways                        | Parameter | Indicator | Data<br>sampling | PROESTOP | REDIM  |
|--------------------|-----------|---|-----------|-----------|------------------|----------|--------|
| Barajas            |           | 3 rapid exits<br>3 right-angle<br>exits | Exits use | p-value   |                  | 1.6%     | 100.0% |
|                    | 610 m     |   | DOTA      | Mean      | 53.9 s           | 54.9 s   | 52.0 s |
| 55                 |           |   | KOTA      | Error     | 0.5%             | -1.9%    | 3.6%   |
| N 1                |           | 2 rapid exits                           | Exits use | p-value   |                  | 2.5%     | 3.2%   |
| Malaga<br>14       | 16 m      |   | ROTA      | Mean      | 62.9 s           | 61.7 s   | 52.0 s |
|                    |           |   |           | Error     | 3.4%             | 1.9%     | 17.4%  |
| Ibiza 06           | 7 m       | 1 rapid exit                            | Exits use | p-value   |                  | 2.6%     | 0.9%   |
|                    |           |   | ROTA      | Mean      | 63.6 s           | 64.2 s   | 61.3 s |
|                    |           |   |           | Error     | 3.4%             | -0.9%    | 3.6%   |
| Menorca<br>19      | 91 m      | right-angle<br>exits                    | Exits use | p-value   |                  | 1.3%     | 7.6%   |
|                    |           |   | ROTA      | Mean      | 68.7 s           | 66.1 s   | 64.8 s |
|                    |           |   |           | Error     | 3.1%             | 3.8%     | 5.6%   |
| Jerez 21           | 25 m      | Apron access<br>No parallel<br>taxiway  | ROTA      | Mean      | 145.8 s          | 141.6 s  |        |
|                    |           |   |           | Error     | 12.3%            | 2.9%     |        |

Table 8. Summary of Validation results at test-bed airports

 $<sup>^{6}</sup>$  In those cases where the p-value of the F-test is less than 0.05, there is a statistically significant difference between the mean of the variable (column) from one level of the factor (row) to another at the 95.0% confidence level.