Prediction of Top of Descent Location for Idle-thrust Descents

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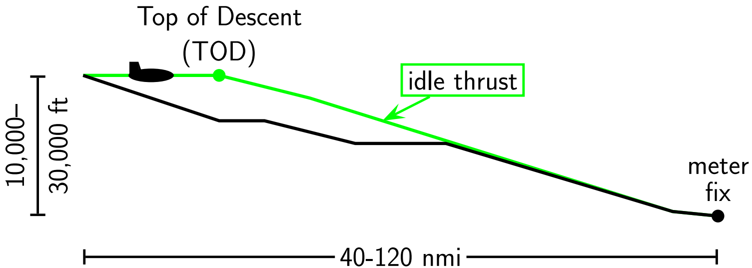
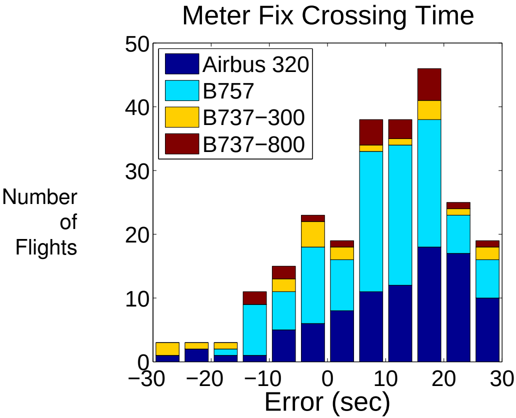
Background

Vertical profile from cruise to meter fix



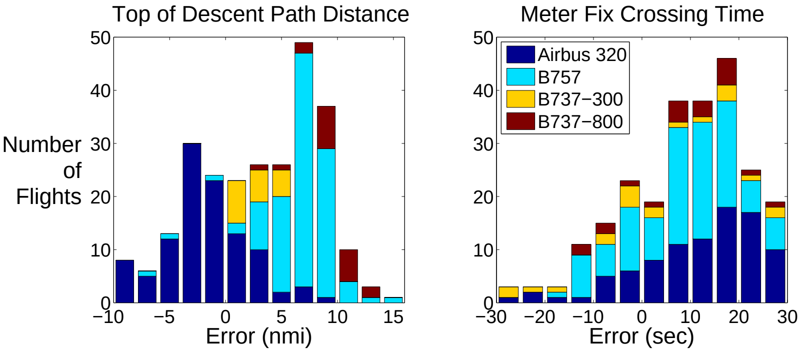
Idle-thrust descent reduces fuel consumption and emissions.

Example Meter Fix Time Errors for Operational Data



- More than 90% of the meter fix crossing time predictions have absolute error less than 30 sec.

Example Prediction Errors for Operational Data



- Less than 50% of the TOD location predictions have absolute error less than 5 nmi.

Outline of Presentation

- Mathematical analysis and approximation of TOD location based on NASA trajectory predictor
- Comparison of results with FMS test bench data and with operational data
- Possible applications

Mathematical Analysis

Treatment of Wind

- For this mathematical analysis, wind is zero
- Incorporating wind into the results is explained in paper
- Analysis of operational data included wind

Kinematic Equations

$$\frac{ds}{dt} = V_t$$
$$\frac{dh}{dt} = \gamma_a V_t$$

t : time
s : the ground path distance relative to the meter fix
h : the altitude
V_t : the true airspeed
γ_a : is the flight path angle with respect to the air mass

Kinetic Equation

$$m\frac{dV_t}{dt} = T - D - mg\gamma_a$$

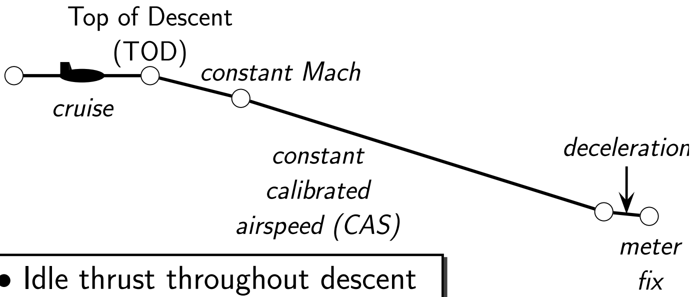
T : thrust
D : drag
m : aircraft mass
g : gravitational acceleration

All Equations for Vertical Profile

$$\frac{1}{g}\frac{dV_t}{dt} = \frac{T - D}{mg} - \gamma_a$$
$$\frac{ds}{dt} = V_t$$
$$\frac{dh}{dt} = \gamma_a V_t$$

Specify 2 out of 3: flight path angle γ_a, airspeed V_t, thrust T

Descent Procedure



- Idle thrust throughout descent
- Known horizontal trajectory

Constant Mach and CAS Segments

$$\frac{1}{g}\frac{dV_t}{dt} = \frac{T - D}{mg} - \gamma_a$$
$$\frac{ds}{dt} = V_t$$
$$\frac{dh}{dt} = \gamma_a V_t$$

$V_t(h)$ specified:

$$\frac{ds}{dh} = \frac{1}{\gamma_a} = \left(1 + \frac{1}{2g}\frac{dV_t^2}{dh}\right)\frac{mg}{T - D}$$

Top of Descent Location

$$S_{\mathrm{TOD}} = -\int_{h_{\mathrm{crz}}}^{h_{\mathrm{fix}}} \left(1 + \frac{1}{2g}\frac{dV_t^2}{dh}\right)\frac{mg}{T-D}\,dh - \frac{1}{g}\int_{V_c}^{V_{\mathrm{fix}}} V_t \frac{mg}{T-D}\,dV_t$$

S_{TOD} : TOD location as path distance relative to meter fix
h_{crz} : cruise altitude
h_{fix} : meter fix altitude
V_c : descent CAS
V_{fix} : meter fix CAS

Top of Descent Location

$$S_{\text{TOD}} = -\int_{h_{\text{crz}}}^{h_{\text{fix}}} \left(1 + \frac{1}{2g}\frac{dV_t^2}{dh}\right) \frac{mg}{T - D}\, dh - \frac{1}{g}\int_{V_c}^{V_{\text{fix}}} V_t \frac{mg}{T - D}\, dV_t$$

Goal

Polynomial approximation in terms of $h_{\text{crz}}, V_c, h_{\text{fix}}, V_{\text{fix}}, W = mg$

Approach

Analyzed trajectories from NASA predictor to develop the polynomial approximations

- Test matrix with 9000 entries varying
  - cruise altitude h_{crz}
  - cruise Mach
  - descent speed V_c
  - meter fix altitude h_{fix}
  - meter fix speed V_{fix}
  - aircraft weight W
- B737-700 and B777-200

Approximation for Constant Mach and Constant CAS

$$\int_{h_{\text{crz}}}^{h_{\text{fix}}} \left(1 + \frac{1}{2g}\frac{dV_t^2}{dh}\right) \frac{W}{T - D} dh$$

$$\approx \mathcal{P}(V_c, W) \int_{h_{\text{crz}}}^{h_{\text{fix}}} \left(1 + \frac{1}{2g}\frac{dV_t^2}{dh}\right) dh$$

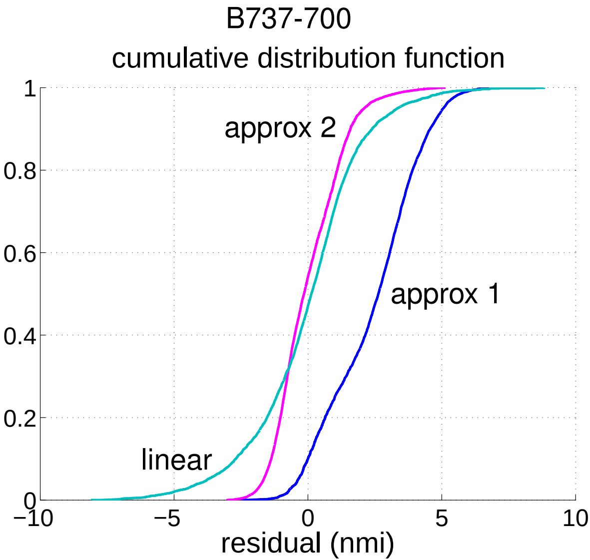
$$\approx -(\Delta h)\mathcal{V}(V_c)\mathcal{P}(V_c, W)$$

$$\approx -(\Delta h) \times [\text{linear function of } V_c, W]$$

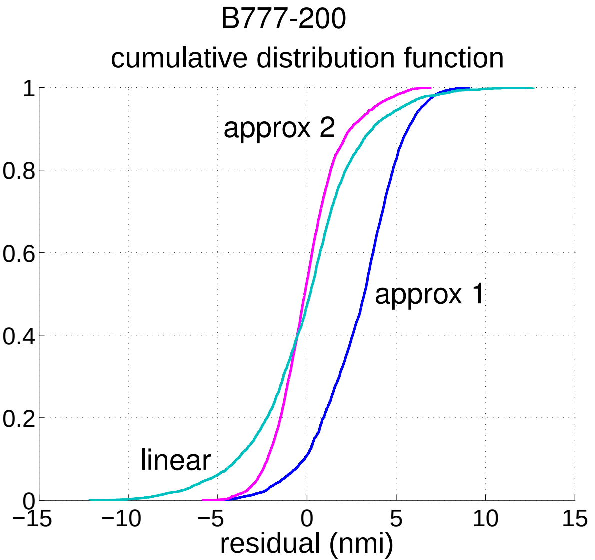
$$\approx \text{linear function of } V_c, W, \Delta h$$

Each of the last three lines gives a polynomial approximation, with decreasing accuracy but increasing simplicity.

Accuracy of Approximations for TOD Location



Accuracy of Approximations for TOD Location



Validation Against Real-World Data

FMS Test Bench Experimental Setup

- Commercial FMS connected to flight simulator
- Same aircraft types as in mathematical analysis
- Test matrix is (3 values of descent CAS V_c) $\times$ (3 values of weight W)
- All other inputs the same in all runs
- Recorded TOD locations computed by FMS

FMS Test Bench Data Results

- Least squares fit with a model linear in V_c and W gave absolute errors less than 2 nmi for both aircraft types
- Much more accurate than expected from previous results
  - Approximations above partly explain this
  - Test matrix does not adequately exercise predictor

Operational Data Collection Procedures

- Collected data for flights arriving at Denver International Airport
- September 8–23, 2009
- United Air Lines and Continental Airlines participated
- Humans in controller facility wrote down:
  - Flights that participated and whether descent interrupted
  - Speed profiles
- Pilots wrote down aircraft weight
- Extracted cruise altitude and horizontal trajectory from radar data

Operational Data Results

- Analyzed about 70 descents each for Airbus 319/320 and B757-200 from one airline
- Least squares fit with a model linear in Δh, V_c, and W gave absolute errors less than 4 nmi for all except:
  - Four (6%) of Airbus descents
  - Six (8%) of B757 descents

Applications

Possible Uses of These Approximations

- Design of FMS test bench experiments
- Simplified sensitivity analysis
- Kinematic predictor of TOD location
  - Dynamic machine learning of coefficients to handle de-icing settings, for example
  - Continuously updated error models
  - Only 5-10 coefficients; NASA predictor has 2000 entries in its table for B777 thrust

More About Kinematic Predictor

- For “what-if” tool with data link
  - Might only consider changing V_c and horizontal trajectory
  - Only need to assume S_{TOD} is linear in V_c
  - Only need to estimate *one* coefficient
- To fill in predicted trajectory between S_{TOD} and meter fix
  - Kinematic approach: straight line might be better than current predictions with 10 nmi error in S_{TOD}
  - Kinetic approach: modify $W/(T - D)$ so that integrator gives S_{TOD} close to desired value
    - Replace with constant
    - Multiply by constant
    - Add constant
    - . . .

Conclusions

- Need to improve prediction of vertical profile to enable fuel-efficient descents in congested airspace
- Largest source of error is difference in $W/(T-D)$ between FMS and ground predictor
- TOD location can be approximated well by a simple function of $h_{\text{crz}}, V_c, h_{\text{fix}}, V_{\text{fix}}, W$
- These approximations might guide research to improve the predictor