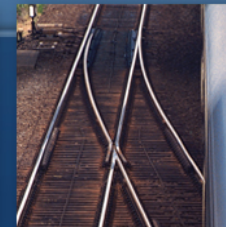
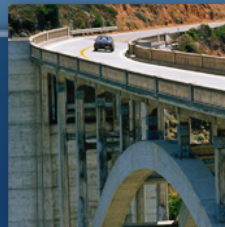


New Method for Probabilistic Traffic Demand Predictions for En Route Sectors Based on Uncertain Predictions of Individual Flight Events

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SERVING THE NATION AS A LEADER IN GLOBAL
TRANSPORTATION INNOVATION SINCE 1970



Outline

- **Introduction/Motivation**
- **Deterministic Sector Demand Predictions**
 - **Current approach**
 - **Problems**
- **Sector Demand Prediction Accuracy**
- **Probabilistic Sector Demand Predictions**



Introduction

- Major functions of Traffic Flow Management System (TFMS):
 - Predict and monitor traffic demand and capacity at National Airspace (NAS) elements for several hours into the future
 - Alert TFM specialists on potential congestion whenever and wherever predicted traffic demand exceeds available capacity
 - Provide decision support to TFM specialists on triggering Traffic Management Initiatives (TMI) for congestion management
- Quality of TFM decisions significantly depends on accuracy of predictions



Accurate Traffic Demand Predictions are Needed...

- For reliable identification of potential congestion (magnitude and duration)
- For better TFM decision making on triggering TMIs
- For optimal planning on utilization of operational resources at airports and in airspace
- For better management of airline resources
- For providing better service to passengers



Deterministic Predictions of Traffic Demand in TFMS

- For each flight, the following information is available
 - Intent (flight plan)
 - Current status (still on the ground or in the air)
 - Current location (tracking)
- TFMS combines this information to project the aircraft position in the future
 - Entry and exit for en route sectors
 - Arrival at the destination airport
- The deterministic flight-by-flight predictions are then rolled up into aggregate demand predictions
- If the aggregate prediction exceeds capacity (the MAP), an alert is triggered



Traffic Demand in Sectors vs. Traffic Demand at Airports

Airports	Sectors
15-minute aggregation	One-minute aggregation
15-minute demand includes all flights with ETA/ETD within a 15-minute interval	The peak one-minute count within a 15-minute interval determines sector demand for entire 15-minute interval
Successive 15-minute intervals contain different flights	Successive minutes contain many of the same flights
Flight arrival times are needed for aggregating flights in 15 minutes	Both flight arrival times and times-in-sector are needed for aggregating flights in one-minute

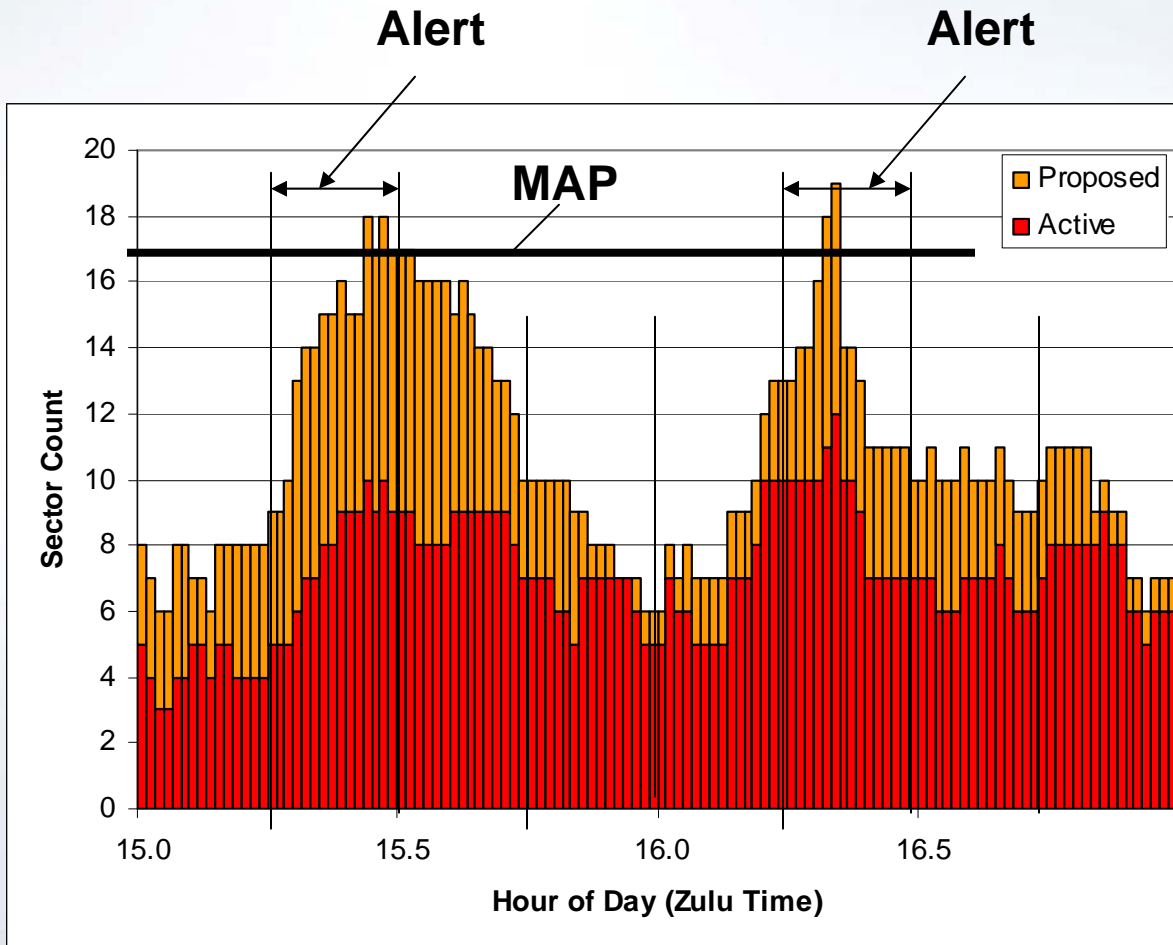


What is Wrong with the Current TFMS Measure of Traffic Demand in En route Sectors?

- Does not take into account uncertainty in predictions
- Rely on one-minute counts
- Use a one-minute peak from fifteen one-minute counts as a traffic demand for entire 15-minute interval
- This measure of traffic demand in sectors
 - is inaccurate
 - is unstable
 - does not adequately reflect sector congestion and workload



One-minute Count Predictions for ZMP20 4/13/2009

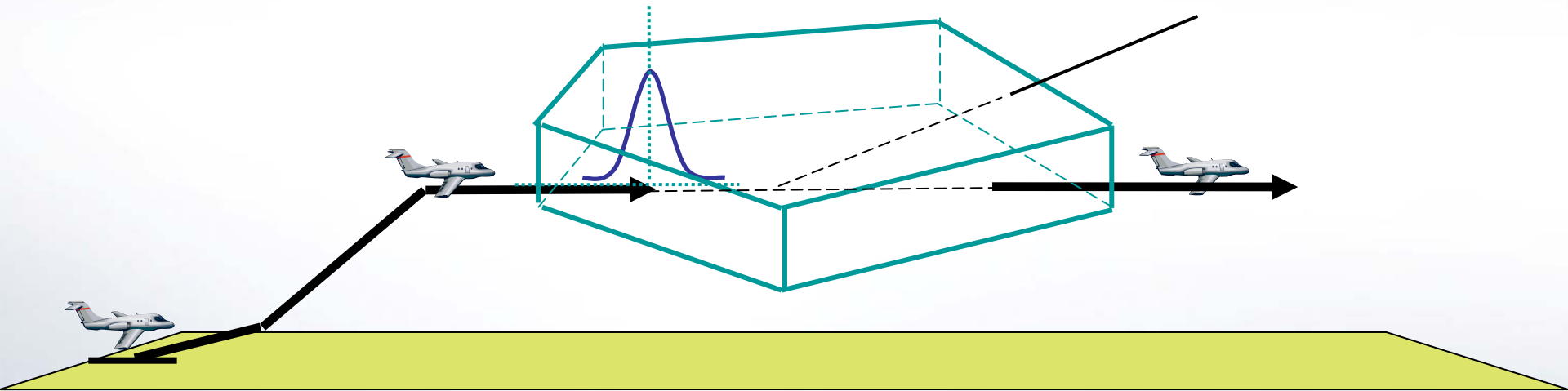


The Goal of the Study

- Analyze uncertainty in predicting times of en route events for individual flights
- Develop an analytical method for probabilistic predictions of traffic demand for en route sectors via transferring the characteristics of uncertainty in predictions for individual flights into characteristics of uncertainty in aggregate demand counts



Accuracy of Flight's Event Time Predictions

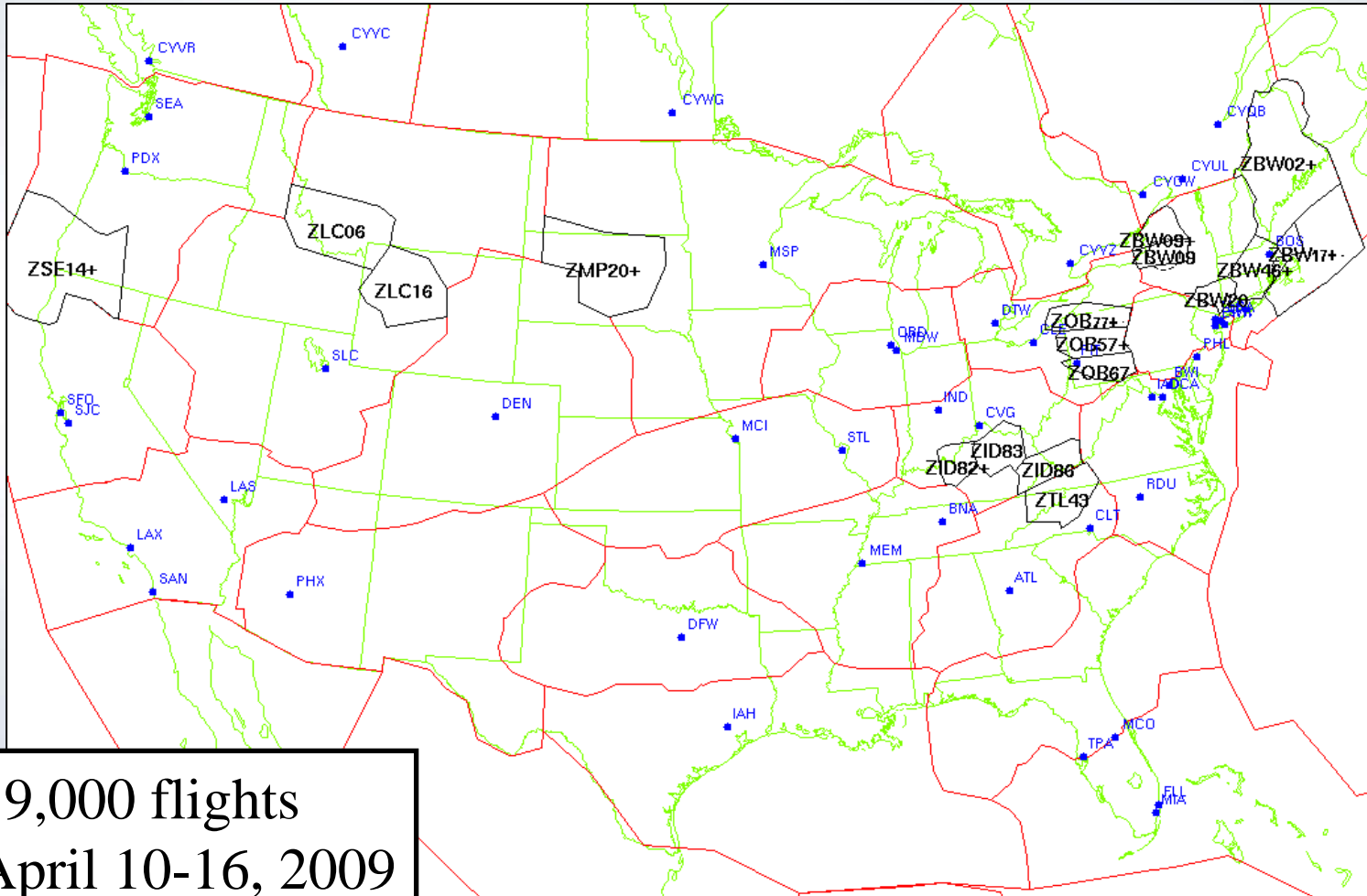


Analysis of Accuracy of Flight's Event Time Predictions

- The analysis was performed on the TFMS data for sixteen en-route sectors: ZBW02, ZBW09, ZBW17, ZBW20, ZBW46, ZID82, ZID83, ZID86, ZLC06, ZLC16, ZMP20, ZOB57, ZOB67, ZOB77, ZSE14, and ZTL43
- Predicted times for flights entering and exiting sectors were collected for different LAT (up to 3 hours) during April 10 – 16, 2009: altogether, the data for 39,000 flights was analyzed
- Analysis was performed separately for active and proposed flights
- Probability distributions of errors were determined for various LATs



Sector Data Examined (16 En route Sectors)

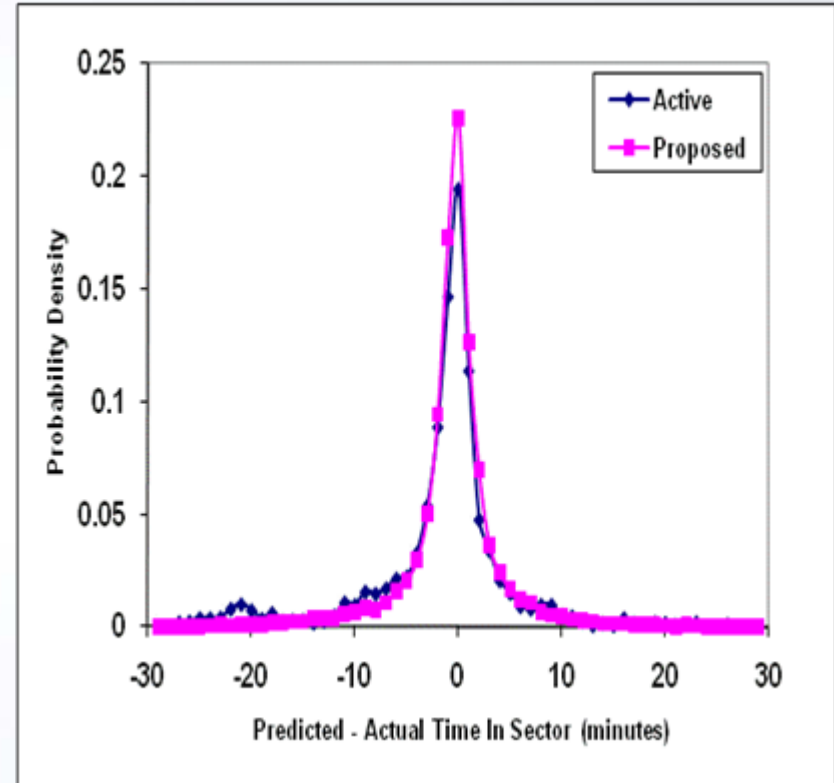
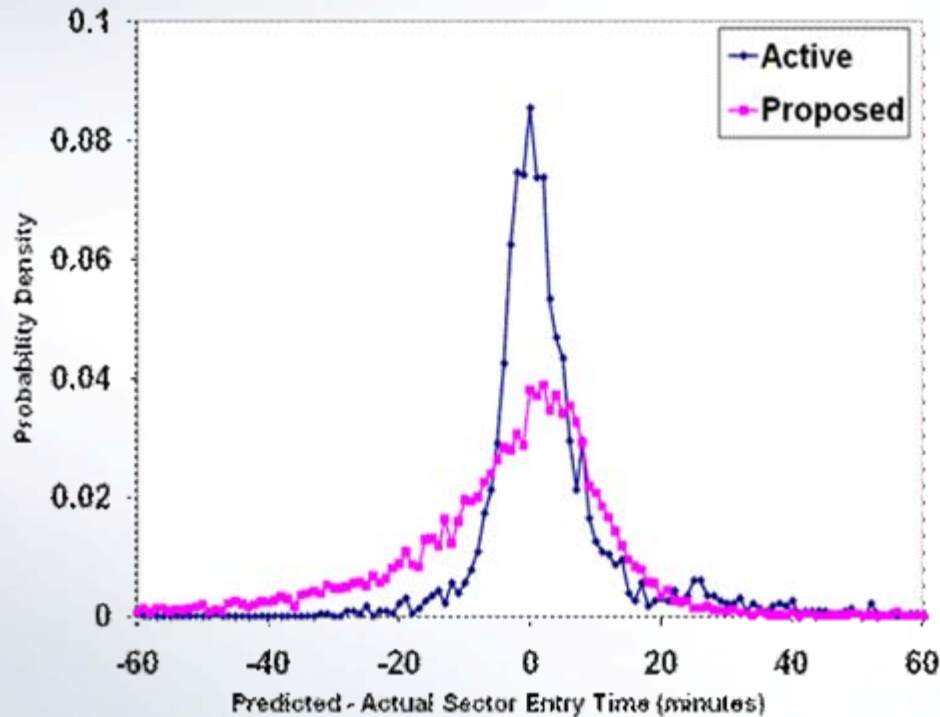


39,000 flights
April 10-16, 2009



Accuracy of Flights' Sector Entry Time and Time in Sector Predictions in TFMS

LAT = 90 min



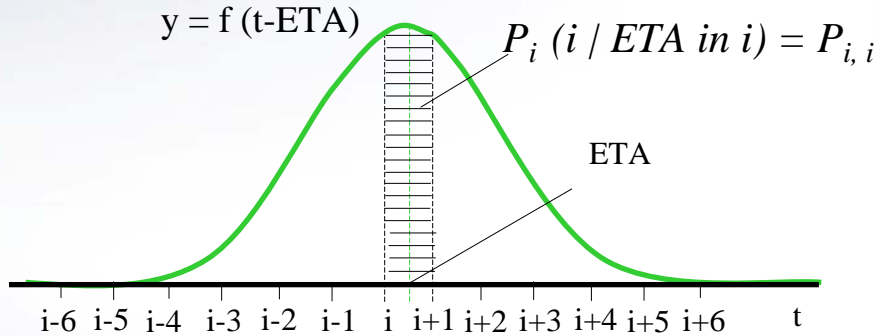
St. deviation: 4 – 12 min for active flights
15 – 22 min for proposed flights

St. deviation: does not exceed 4 min for
both active and proposed flights

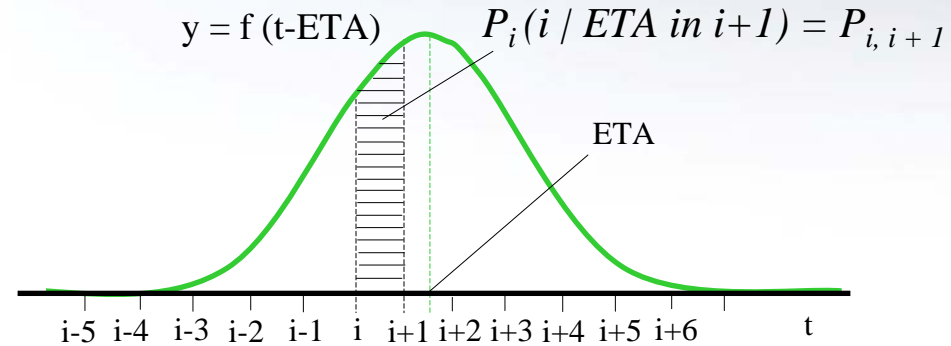


Probability for a Flight to Enter a Sector at a 1-minute Interval

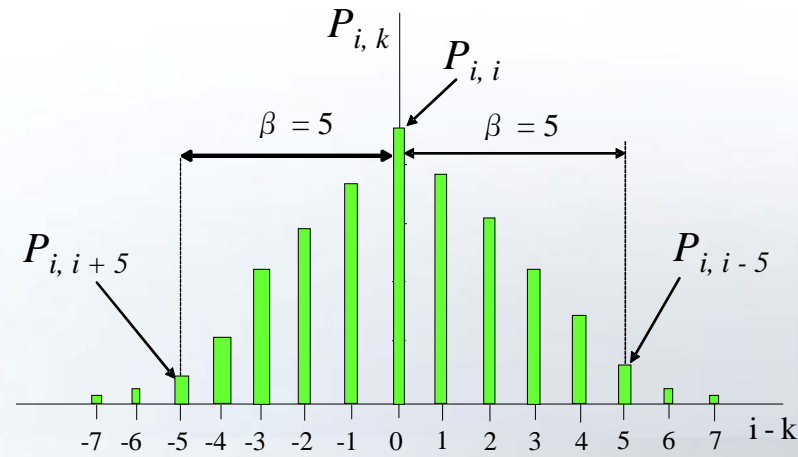
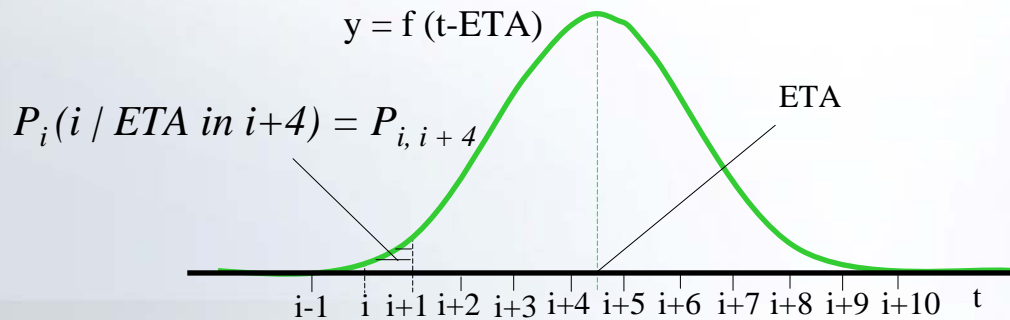
ETA within a 1-minute interval of interest



ETA outside of a 1-minute interval of interest



ETA is far from a 1-minute interval of interest



Notation

d_k – number of flights deterministically predicted to enter a sector during a one-minute interval k

$d_k = d_k^{(a)} + d_k^{(g)}$, where

$d_k^{(a)}$ – number of airborne flights in the demand d_k

$d_k^{(g)}$ – number of proposed flights in the demand d_k

$F(x)$ – a cumulative distribution function (CDF) of prediction error in sector entry time

$F^{(a)}(x)$ – a CDF of prediction error in sector entry time for active flights

$F^{(g)}(x)$ – a CDF of prediction error in sector entry time for proposed flights

$P_{i,k}$ – a probability for a flights deterministically predicted to enter a sector during interval k to enter a sector during interval i

$P_{i,k} \approx 0.5[F(i-k+1) + F(i-k-1)]$



Distribution of Number of Flights Entering a Sector at a Specific One-minute Interval

$d_{i,k}$ – a random number of flights entering a sector during interval of interest i from a set of flights d_k predicted to enter a sector during interval k

$P_i^{(k)}(x)$ – probability for x flights from the set of d_k flights to enter a sector during interval i

$P_i^{(k)}(x) = C_x^{d_k} P_{i,k}^x (1-P_{i,k})^{(d_k-x)}$, $x = 0, 1, 2, \dots, d_k$; - Binomial distribution

$$C_x^{d_k} = \frac{d_k!}{x!(d_k-x)!}, \quad x! = 1*2* \dots * (x-1)* x; \quad 0! = 1$$

The expected (mean) number of flights from d_k to enter a sector during interval i :

$$\overline{d_{i,k}} = P_{i,k} d_k$$

The standard deviation of number of flights from d_k to enter a sector during interval i :

$$\sigma_{i,k} = \sqrt{P_{i,k}(1-P_{i,k})d_k}$$



Example:

very low probability for all deterministically predicted flights to enter a sector at the interval they predicted to enter to

Probability for all flights d_i deterministically predicted to enter a sector during interval i to enter a sector during this interval:

$$P_i^{(i)}(d_i) = P_{i,i}^{d_i}$$

$$P_{i,i} = 0.1; d_i = 10$$

$$P_i^{(i)}(d_i) = 0.1^{10} \approx 0$$

$$\overline{d_{i,i}} = P_{i,i} d_i = 1$$



Probabilistic Predictions of a Number of Flights Entering a Sector during a One-minute Interval

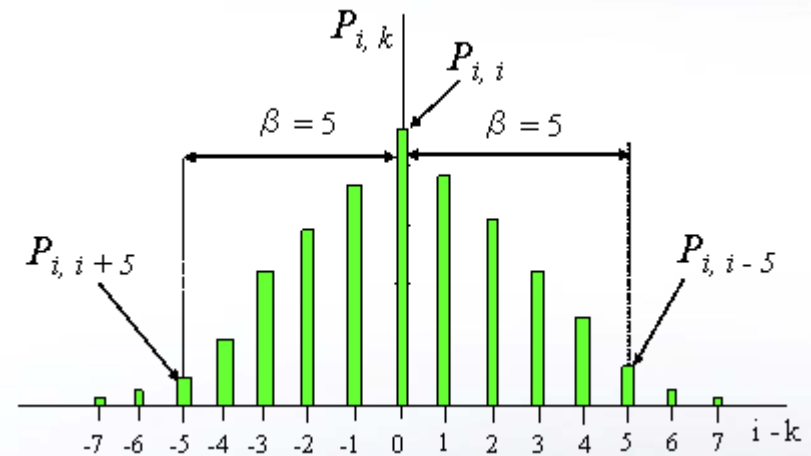
Random number of flights entering a sector during interval i :

$$\tilde{d}_i = \sum_k d_{i,k}$$

Expected (mean) number of flights entering a sector during interval i :

$$\bar{d}_i = \sum_k P_{i,k} d_k$$

$$\bar{d}_i \approx \sum_{k=i-\beta}^{i+\beta} P_{i,k} d_k$$

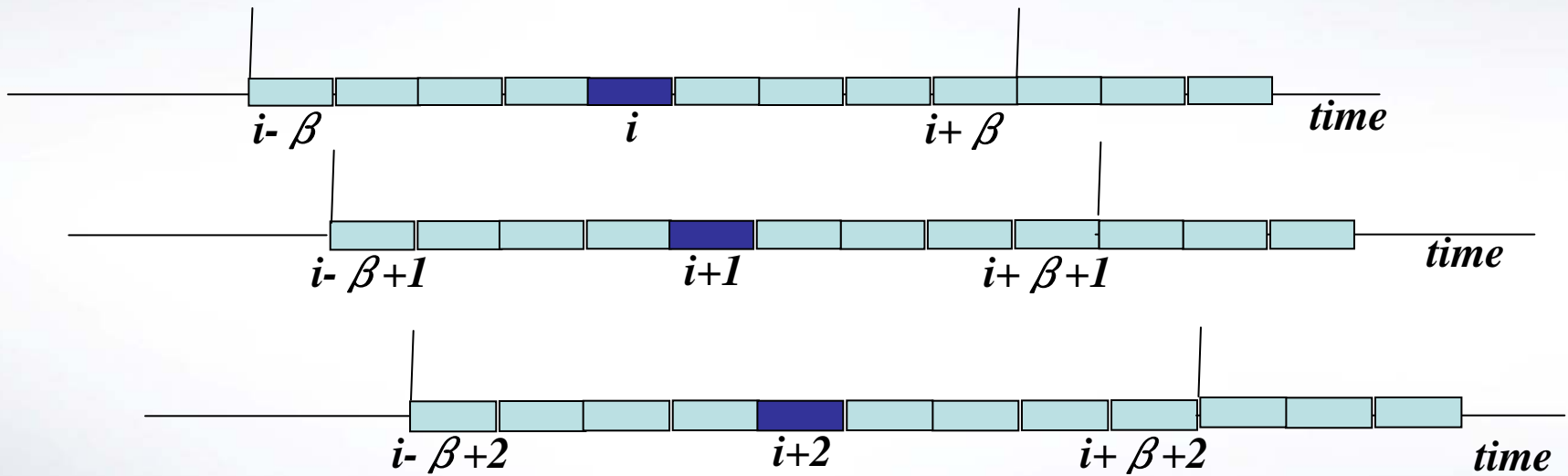


Standard deviation of number of flights entering a sector during interval i :

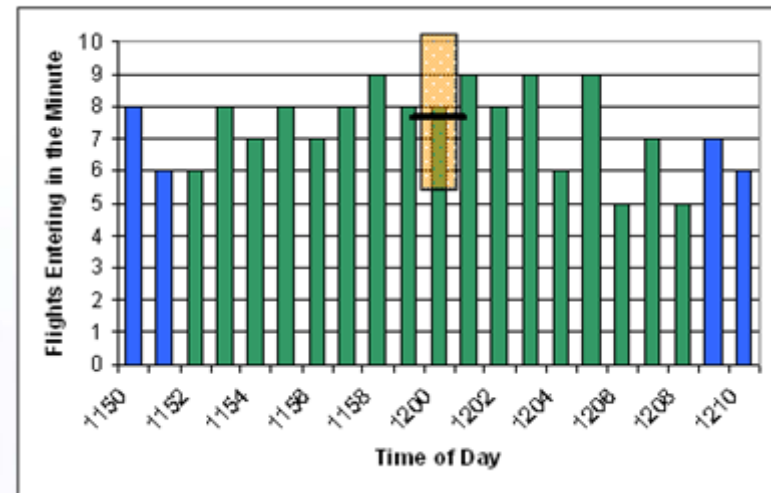
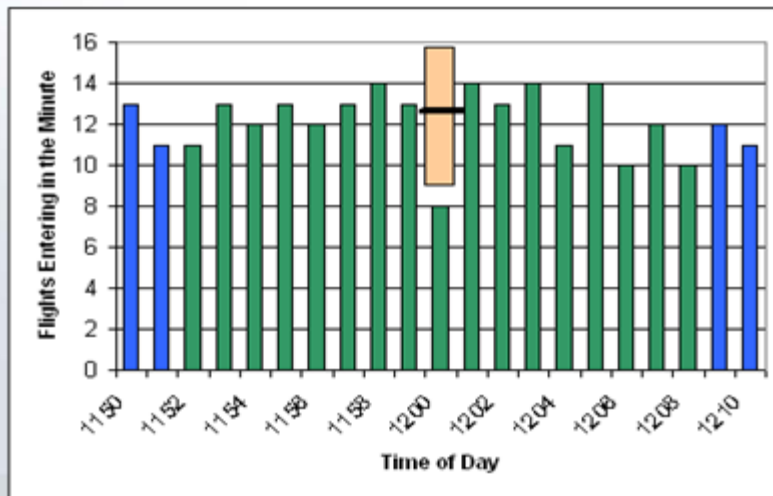
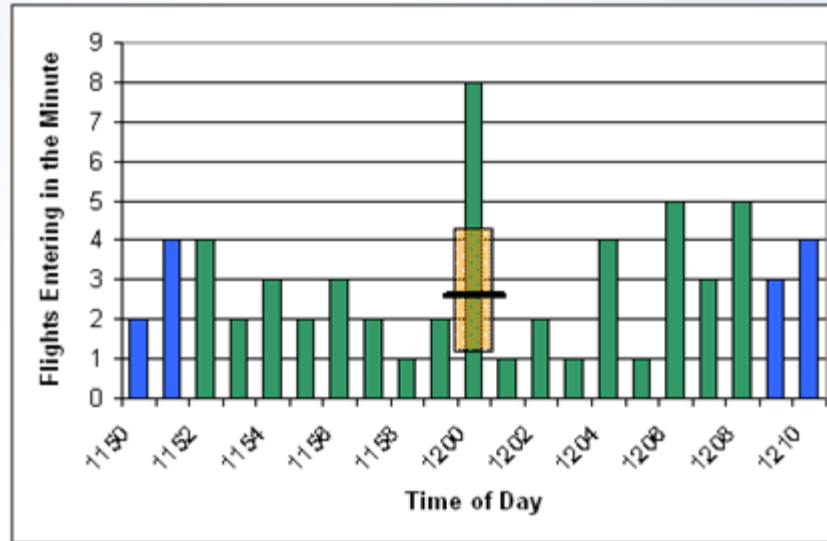
$$\sigma_i \approx \sqrt{\sum_{k=i-\beta}^{i+\beta} P_{i,k} (1-P_{i,k}) d_k}$$



Sliding Window



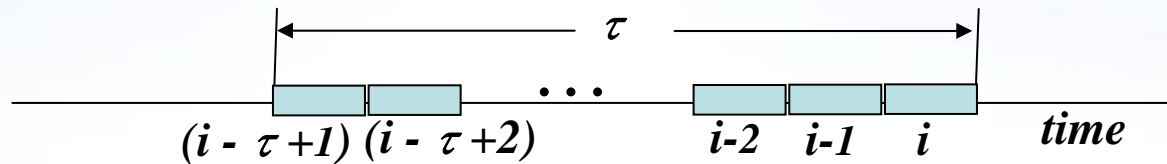
Examples



Probabilistic Prediction of One-minute Traffic Demand in a Sector

In addition to flight's sector entry time, a predicted flight's time in a sector is needed.

Basic Case: all flights need the same time τ to traverse the sector (same time-in-sector)



Deterministic prediction of the number of flights $D_{i, \tau}$ in a sector during one-minute interval i is a sum of deterministically predicted number of flights entering the sector during interval i and $(\tau - 1)$ consecutive intervals preceding the interval i

$$D_{i, \tau} = d_i + d_{i-1} + d_{i-2} + \dots + d_{i-(\tau-2)} + d_{i-(\tau-1)} = \sum_{j=i-\tau+1}^i d_j$$

Random number of flights predicted to be in a sector during one-minute interval i is a sum of random number of flights entering the sector during interval i and $(\tau - 1)$ consecutive intervals preceding interval i

$$\tilde{D}_{i, \tau} = \sum_{j=i-\tau+1}^i \tilde{d}_j$$



Probabilistic Prediction of One-minute Traffic Demand in a Sector (cont.)

Expected number of flights:

$$\bar{D}_{i, \tau} = \sum_{j=i-\tau+1}^i \bar{d}_j; \quad \bar{D}_{i, \tau} = \sum_{j=i-\tau+1}^i \sum_{k=j-\beta}^{j+\beta} P_{j, k} d_k$$

$$\bar{D}_{i, \tau} = \sum_{k=i-\tau-\beta+1}^{i+\beta} P_{i, k, \tau} d_k$$

$$P_{i, k, \tau} = \sum_{j=\max(k-\beta, i-\tau+1)}^{\min(i, k+\beta)} P_{j, k}$$

where $i - \beta - \tau + 1 \leq k \leq i + \beta$

Standard deviation of number of flights:

$$\sigma(\bar{D}_{i, \tau}) = \sqrt{\sum_{k=i-\beta}^{i+\beta} P_{i, k, \tau} (1 - P_{i, k, \tau}) d_k}$$

- $P_{i, k, \tau}$ is a probability for a flight to be in a sector during a one-minute interval i if its ETA to enter a sector is in interval k and time for traversing sector is τ .
- For $P_{i, k, \tau}$ this summation states that:
 - the relevant values of j are those ranging from $i - \tau + 1$ to i , and
 - we are only interested in those values of j that are within β of k .



Extension 1:

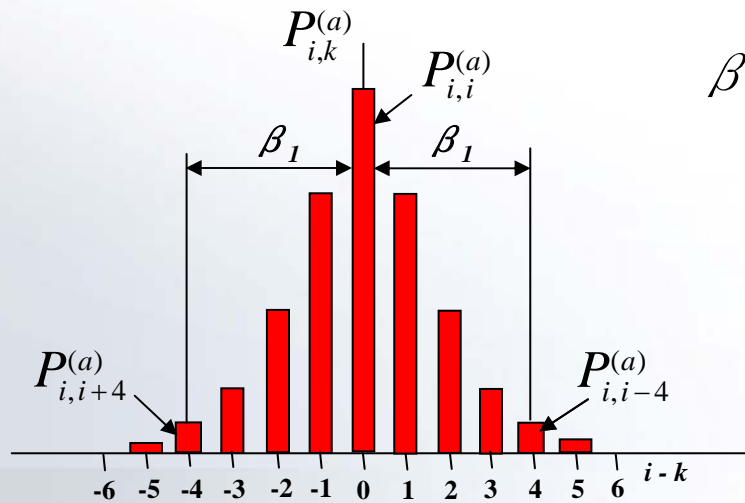
Active and Proposed Flights in Predicted Sector Demand

The number of flights deterministically predicted to enter a sector during the k^{th} one minute interval may include both active and proposed flights:

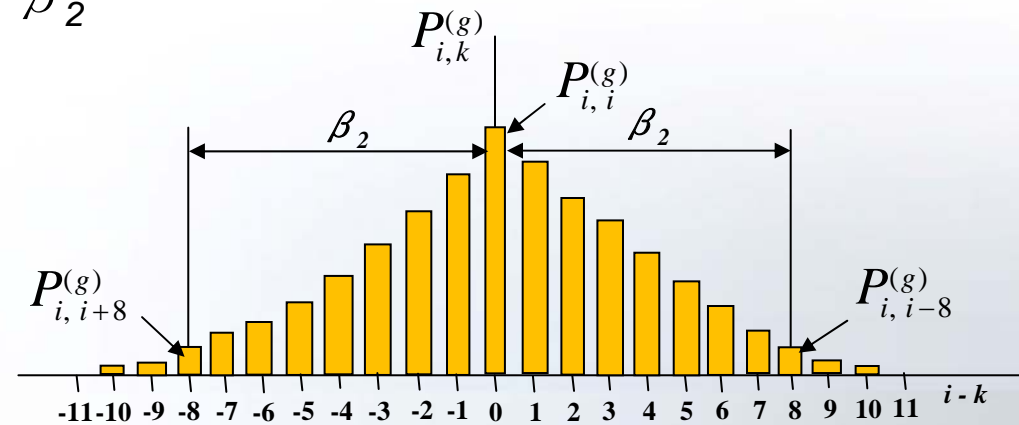
$$d_k = d_k^{(a)} + d_k^{(g)},$$

$F^{(a)}(x)$ and $F^{(g)}(x)$ – are CDF of prediction error in sector entry time for active and proposed flights, respectively

$$P_{i,k}^{(a)} \approx 0.5[F^{(a)}(i-k+1) + F^{(a)}(i-k-1)]; \quad P_{i,k}^{(g)} \approx 0.5[F^{(g)}(i-k+1) + F^{(g)}(i-k-1)]$$



$$\beta_1 < \beta_2$$



Active and Proposed Flights in Predicted Sector Demand: (cont.)

All flights have the same time τ for traversing a sector

Expected number of flights in a sector during a one-minute i :

$$\overline{\tilde{D}}_{i, \tau} = \sum_{k=i-\tau-\beta_1+1}^{i+\beta_1} P_{i,k,\tau}^{(a)} d_k^{(a)} + \sum_{k=i-\tau-\beta_2+1}^{i+\beta_2} P_{i,k,\tau}^{(g)} d_k^{(g)}$$

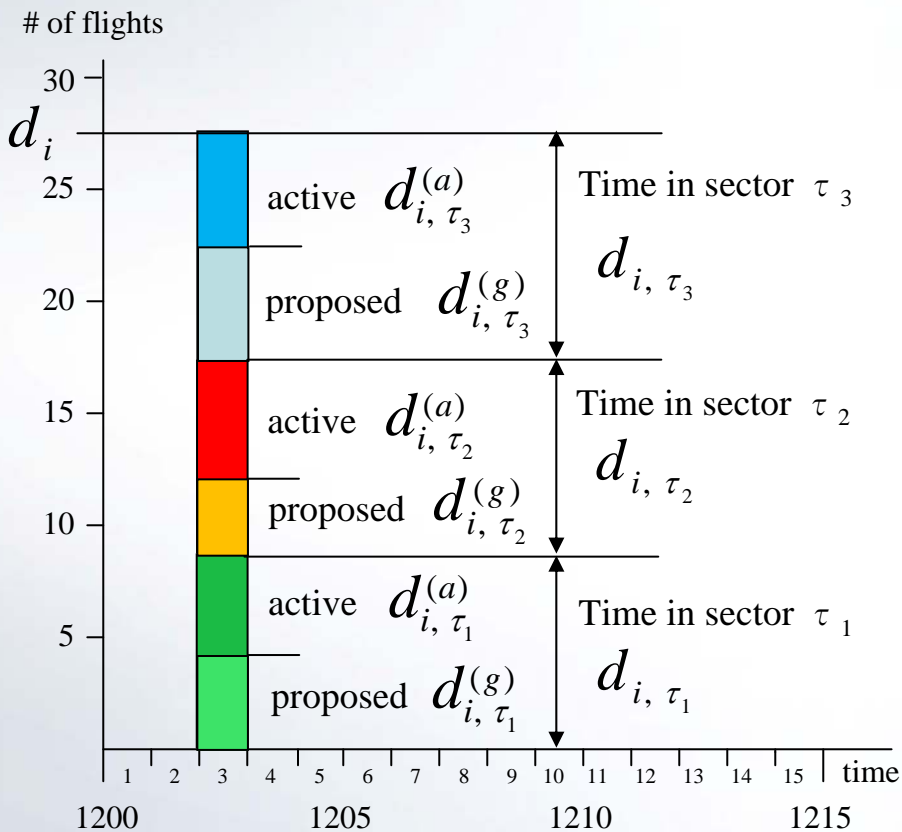
Variance of number of flights in a sector during a one-minute i :

$$\text{Var}(\tilde{D}_{i, \tau}) = \sum_{k=i-\tau-\beta_1+1}^{i+\beta_1} P_{i,k,\tau}^{(a)} (1 - P_{i,k,\tau}^{(a)}) d_k^{(a)} + \sum_{k=i-\tau-\beta_2+1}^{i+\beta_2} P_{i,k,\tau}^{(g)} (1 - P_{i,k,\tau}^{(g)}) d_k^{(g)}$$

$$P_{i,k,\tau}^{(a)} = \sum_{j=\max(k-\beta_1, i-\tau+1)}^{\min(i, k+\beta_1)} P_{j,k}^{(a)} ; \quad P_{i,k,\tau}^{(g)} = \sum_{j=\max(k-\beta_2, i-\tau+1)}^{\min(i, k+\beta_2)} P_{j,k}^{(g)}$$



Extension 2: Flights Have Different Times in Sector



Flights might require different times for traversing a sector.

Suppose that at each minute there might be up to m sub-groups of flights entering a sector with different times in a sector:

$$\tau_1, \tau_2, \dots, \tau_m.$$

d_i - number of flights predicted to enter a sector during one-minute i .

$d_{i, \tau_j}^{(a)}$ and $d_{i, \tau_j}^{(g)}$ are active and proposed fractions of demand d_i , respectively, with time in sector τ_j ($j = 1, 2, \dots, m$).



Flights Use Different Times in Sector (cont.)

Active and proposed components at each sub-group of traffic demand:

$$d_{i, \tau_j} = d_{i, \tau_j}^{(a)} + d_{i, \tau_j}^{(g)}, \quad j = 1, 2, \dots, m$$

Expected number of flights in a sector during a one-minute i :

$$\bar{\tilde{D}}_i = \sum_{j=1}^m \left(\sum_{k=i-\tau_j-\beta_1+1}^{i+\beta_1} P_{i, k, \tau_j}^{(a)} d_{k, \tau_j}^{(a)} + \sum_{k=i-\tau_j-\beta_2+1}^{i+\beta_2} P_{i, k, \tau_j}^{(g)} d_{k, \tau_j}^{(g)} \right)$$

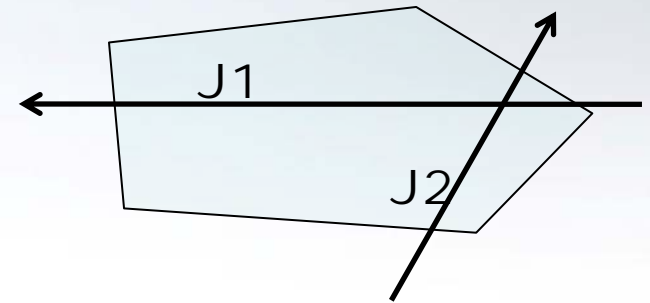
Variance of number of flights in a sector during a one-minute i :

$$\text{Var}(\tilde{D}_i) = \sum_{j=1}^m \left[\sum_{k=i-\tau_j-\beta_1+1}^{i+\beta_1} P_{i, k, \tau_j}^{(a)} (1 - P_{i, k, \tau_j}^{(a)}) d_{k, \tau_j}^{(a)} + \sum_{k=i-\tau_j-\beta_2+1}^{i+\beta_2} P_{i, k, \tau_j}^{(g)} (1 - P_{i, k, \tau_j}^{(g)}) d_{k, \tau_j}^{(g)} \right]$$



Extension 3: Flights Use Different Routes in Sector

Suppose a sector has r routes each of which requires time τ_j ($j = 1, 2, \dots, r$) for a flight to traverse a sector.



One-minute traffic demand counts deterministically predicted to enter a sector (to cross sector boundary) can be sorted out by several sub-groups of flights each of which is associated with a specific route:

$d_i = \sum_{j=1}^r d_{i, \tau_j}$ - number of flights predicted to enter a sector during one-minute i

d_{i, τ_j} - a fraction of demand d_i that uses route j with the time in sector τ_j ($j = 1, 2, \dots, r$)



Flights Use Different Routes in Sector (cont.)

Active and proposed components at each sub-group of traffic demand:

$$d_{i, \tau_j} = d_{i, \tau_j}^{(a)} + d_{i, \tau_j}^{(g)}, \quad j = 1, 2, \dots, r$$

Expected number of flights in a sector during a one-minute i :

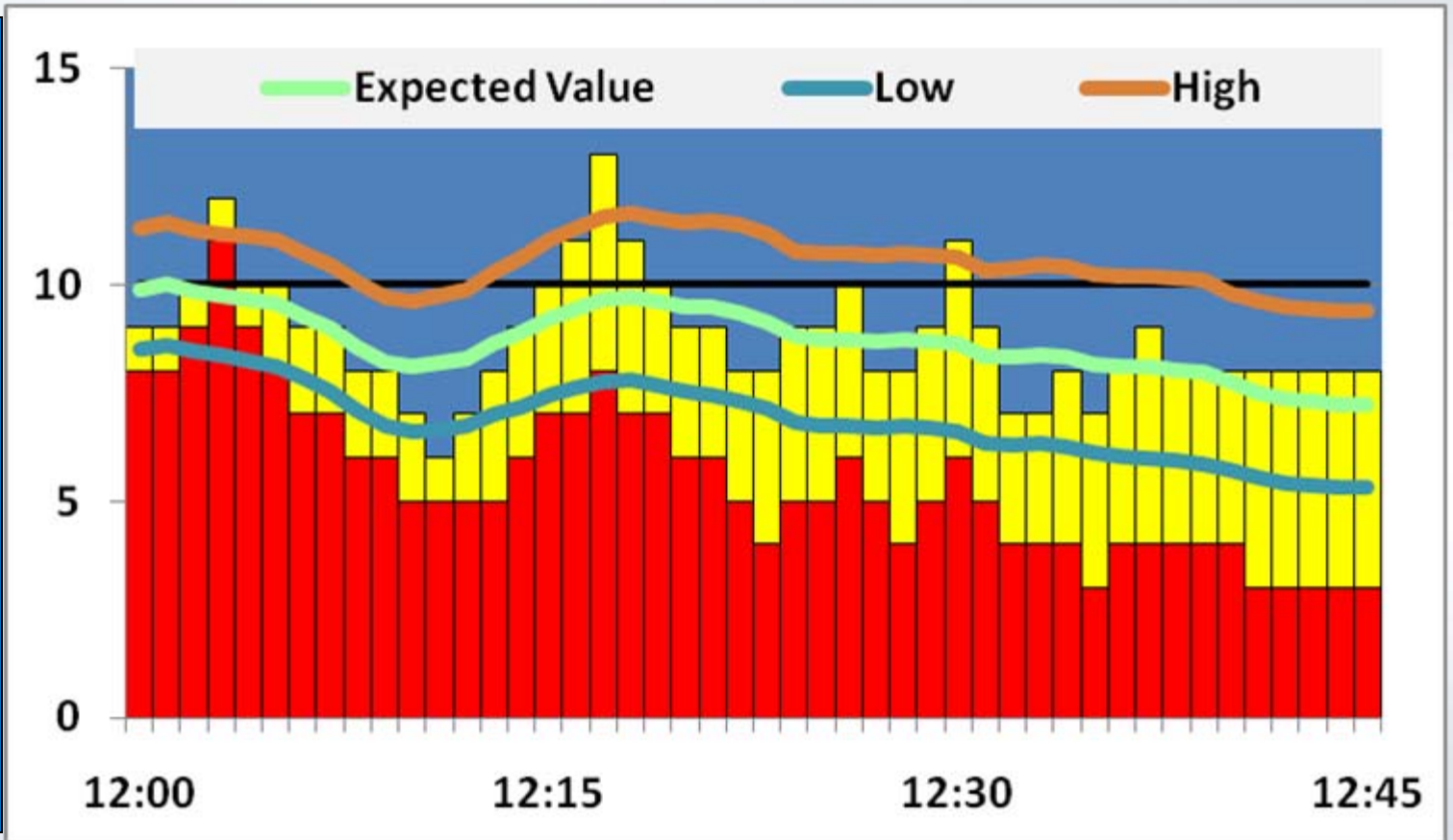
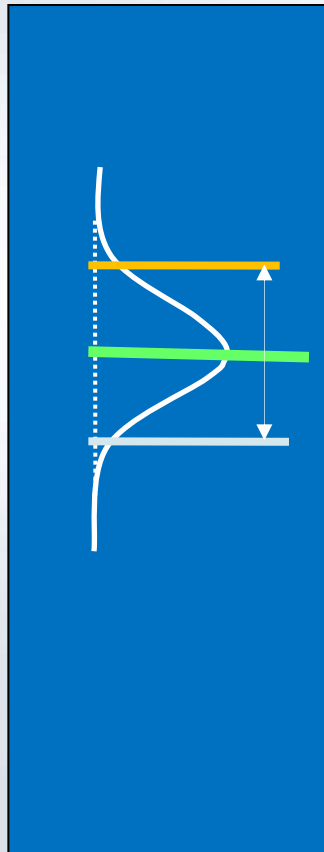
$$\bar{\tilde{D}}_i = \sum_{j=1}^r \left(\sum_{k=i-\tau_j-\beta_1+1}^{i+\beta_1} P_{i, k, \tau_j}^{(a)} d_{k, \tau_j}^{(a)} + \sum_{k=i-\tau_j-\beta_2+1}^{i+\beta_2} P_{i, k, \tau_j}^{(g)} d_{k, \tau_j}^{(g)} \right)$$

Variance of number of flights in a sector during a one-minute i :

$$\text{Var}(\tilde{D}_i) = \sum_{j=1}^r \left[\sum_{k=i-\tau_j-\beta_1+1}^{i+\beta_1} P_{i, k, \tau_j}^{(a)} (1 - P_{i, k, \tau_j}^{(a)}) d_{k, \tau_j}^{(a)} + \sum_{k=i-\tau_j-\beta_2+1}^{i+\beta_2} P_{i, k, \tau_j}^{(g)} (1 - P_{i, k, \tau_j}^{(g)}) d_{k, \tau_j}^{(g)} \right]$$



Probabilistic Predictions



Conclusion

- **New analytical method has been developed to quantify uncertainty in traffic demand predictions for sectors, which is crucial for strategic congestion management in airspace.**
- **The method translates characteristics of uncertainty in predictions of times for individual flight events into characteristics of uncertainty in aggregate traffic demand predictions for sectors.**
- **The whole approach is based on using deterministic predictions of sector entry counts along with probability distributions of errors in predicting sector entry times for active and proposed flights, as well as flights' times-in-sector.**
- **The analytical results can be used for quantifying effects of future improvements in accuracy of timing for individual flights' predictions on improvements in accuracy of aggregate traffic demand predictions**

